Bachelor of Science (PCM)
(B.Sc. - PCM)

Wave Mechanics
(DBSPCO102T24)

Self-Learning Material
(SEM 1)

Jaipur National University
Centre for Distance and Online Education

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PREFACE

A comprehensive mathematical theory of oscillations and waves in real systems is developed in Oscillations and Waves: An Introduction. The author emphasizes physics over mathematics and uses several examples from electronic circuits, electromagnetic waves, continuous gases, fluids, and elastic materials, as well as discrete mechanical, optical, and quantum mechanical systems.

This book assumes some knowledge of college-level mathematics and physics laws. It concentrates on waves and oscillations with linear differential equations. The author discusses areas of optics including wave optics that critically rely on light's wave-like properties. This enables students to fully comprehend the representation of waves and oscillations in terms of normal trigonometric functions before utilizing the more practical but abstracted complicated form.

This tried-and-true textbook, which is based on the author's long-running course at the Jaipur National University at Jaipur, aids students in developing a solid physical grasp of wave phenomena. It facilitates the challenging shift that students experience when moving from lower-division courses that primarily deal with algebraic equations to upper-division courses that focus on differential equations.

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Chapter 1

Superposition of Harmonic Oscillations

Objectives

1. Studying the superposition of harmonic oscillations, along with the principles of linearity and the superposition principle.
2. By investigating the superposition of collinear oscillations with equal and different frequencies.
3. Examining the phenomenon of beats, and extending the concept to N collinear harmonic oscillations

1.1.Principle of superposition

According to the superposition principle, "the vector sum of the individual displacements is the resultant of two or more harmonic displacements."

Any two answers added together constitute a solution for a linear homogeneous differential equation.

Consider a linear homogeneous differential equation of degree n:

$$a_n \frac{d^n y}{dt^2} + a_{n-1} \frac{d^n y}{dt^2} + ------ + a_1 \frac{dy}{dt} + a_0 y = 0$$

This equation has two solutions, y_1 and y_2, hence y_1+y_2 is also a solution.

In case of forced oscillator:

$$m\frac{d^2x}{dt^2} = -kx + F(t)$$

Here the driven force $F(t)$ which is independent of x. Suppose $F_1(t)$ and $F_2(t)$are two driven force, produces oscillations $x_1(t)$ and $x_2(t)$ respectively. If net driven force are $F_1(t)+F_2(t)$, there corresponding oscillation is represented by

$$x(t) = x_1(t) + x_2(t)$$

1.2.Two collinear SHMs with the same frequency but distinct amplitudes and phases superposed

Displacement equation of two different SHMs represented by:

$$x_1(t) = a_1 \cos(\omega t + \delta_1) \quad (1)$$

$$x_2(t) = a_2 \cos(\omega t + \delta_2) \quad (2)$$

Here $a_1$and a_2 are amplitudes and the initial phase angle of two SHMs are δ1 and δ2 which having same angular frequency ω.

1.3.Analytical method

The resultant displacement after superposition principle is represented by:

$$x(t) = x_1(t) + x_2(t)$$

$$x(t) = a_1 \cos(\omega t + \delta_1) + a_2 \cos(\omega t + \delta_2)$$

$$x(t) = (a_1 \cos\delta_1 + a_2 \cos\delta_2)\cos\omega t + (a_1 \sin\delta_1 + a_2 \sin\delta_2)\sin\omega t \quad (3)$$

Putting

$$a_1 \cos\delta_1 + a_2 \cos\delta_2 = A\cos\varphi \quad (4)$$

$$a_1 \sin\delta_1 + a_2 \sin\delta_2 = A\sin\varphi \quad (5)$$

We get

$$x = A\cos(\omega t + \varphi) \quad (6)$$

It demonstrates that the motion is always harmonious. Equations 4 and 5 can be used to get the constants A and φ. When we square and combine equation 4 and 5, we obtain

$$A^2 = a_1^{\ 2} + a_2^{\ 2} + 2a_1a_2\cos(\delta_1 + \delta_2)$$

Dividing equation 5 by 4, we get

$$\tan\varphi = \frac{\mathrm{a_1}\sin\delta_1 + \mathrm{a_2}\sin\delta_2}{\mathrm{a_1}\cos\delta_1 + \mathrm{a_2}\cos\delta_2}$$

The resultant motion is also simple harmonic, its displacement equation is

$$x = A\cos(\omega t + \varphi)$$

1.4.Vector method

The resultant of SHMs with the same frequency can be easily obtained using the rotating vector representation of SHM. Let's use a rotating vector OB_1 of constant length a_1 to represent the first SHM. It will rotate anticlockwise at a constant angular velocity ω, forming an angle $\omega t+\delta_1$ with the x-axis at each given time t.

The displacement x_1 at every time t is given by the projection ON_1 of this vector on the x-axis. Likewise, the second SHM will be represented by the rotating vector OB_2.

Now, the vector sum of OB_1 and OB_2 will provide the resulting motion. The magnitude A of the resulting OB is determined by the parallelogram of vector addition, which is

$$A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos(\delta_1 - \delta_2)$$

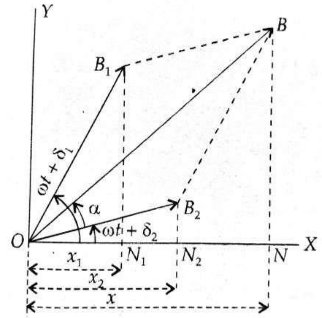


Figure 1: The resulting vector and two vectors that indicate SHMs.

No we get δ = δ2 + α, if the resulting OB has an angle with the x-axis of ωt + δ.

$$\therefore \tan\delta = \frac{\tan\delta_2 + \tan\alpha}{1 - \tan\delta_2 \tan\alpha} \qquad (7)$$

Now

$$\tan\alpha = \frac{a_1\sin(\delta_1 - \delta_2)}{a_2 + a_1\cos(\delta_1 - \delta_2)} \qquad (8)$$

Put the value of tan α from equation 8 in the equation 7, after simplification

$$\tan\alpha = \frac{a_1\sin\delta_1 + a_2\sin\delta_2}{a_1\cos\delta_1 + a_2\cos\delta_2}$$

Projection of $\overrightarrow{OB}$ on x axis represents the simple harmonic motion

$$x = A\cos(\omega t + \delta)$$

Two SHMs operating in the same direction but at rather different frequencies: Beats

Let us take two SHMs with angular frequencies, ω and ω + Δω, where Δω << ω.

$$x_1 = a_1\cos(\omega t + \delta_1)$$

$$x_2 = a_2\cos[(\omega + \Delta\omega)t + \delta_2]$$

$$= a_2\cos(\omega t + \delta_2^r)$$

Where $\delta_2^r = \Delta\omega t + \delta_2$.

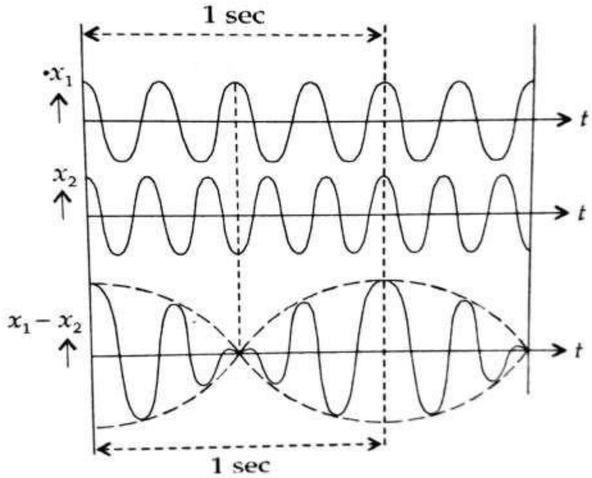


Figure 2: SHMs superposed at marginally different frequencies

The resultant displacement represented by

$$x = x_1 + x_2$$

$$x = (a_1\cos\delta_1 + a_2\cos\delta_2{}^{r})\cos\omega t - (a_1\sin\delta_1 + a_2\sin\delta_2{}^{r})\sin\omega t$$

We call this phenomena beats. It is occurs when two tuning forks or other sources of sound with almost similar frequencies are played simultaneously. The difference in frequency difference of two individual waves is equal to the number of beats per second. Figure 2 shows the graphical representation of two SHMs wave's superposition. In over case the frequencies of the two waves are 4Hz and 5Hz respectively having same amplitude and same initial phase. In the resultant pattern, the difference between the two successive maxima or minima is 1 sec. Although the amplitude of the resultant wave is also varies with frequency of 1Hz that is similar to the difference of frequencies of the component vibrations.

1.5. Superposition of SHMs*:*

Consider the preliminary conditions corresponds to velocities and amplitudes, the net displacement, results from two or more harmonic displacements that is always a arithmetical summation of the individual displacements. The superposition principle is applicable forso many simple harmonic oscillations. In two dimensional, the directions of such waves are same or perpendicular.

SHM differential equation is:

$$\frac{d^2x}{dt^2} = -\omega_0{}^2 x \qquad (1)$$

It is linear homogenous differential equation of the order two. Such type of equation has considerable property; summation of its two linearly independent solutions is also a solution.

Consider that, $x_1(t)$ and $x_2(t)$ be the solution of two separate linear homogenous differential equations

$$\frac{d^2 x_1}{dt^2} = -\omega_0{}^2 x_1 \tag{2}$$

$$\frac{d^2 x_2}{dt^2} = -\omega_0{}^2 x_2 \tag{3}$$

The resultant equation by adding equation 1 and 2 is

$$\frac{d^2 (x_1 + x_2)}{dt^2} = -\omega_0{}^2 (x_1 + x_2) \tag{4}$$

On the basis of superposition principle, the resultant is

$$x(t) = x_1(t) + x_2(t) \tag{5}$$

Equation 5 also satisfies the equation 1. It proves that, It illustrates that the same linear homogeneous differential equation, satisfied separately by x_1 and x_2, also satisfied by x(t) superposition of two displacements equations.

Two identically harmonic oscillations superposed travelling along the same line:

Let us consider two collinear harmonic oscillations of different amplitudes (a_1 and a_2) and same frequency ω_0 and differ in phase by π. Displacements equation for these oscillations are represented as:

$$x_1(t) = a_1 \cos(\omega_o t) \tag{6}$$

$$x_2(t) = a_2 \cos(\omega_o t + \pi)$$

$$x_2(t) = -a_2 \cos(\omega_o t) \tag{7}$$

From the superposition principle, resulting displacement is given by

$$x(t) = x_1(t) + x_2(t)$$

$$x(t) = a_1 \cos(\omega_o t) - a_2 \cos(\omega_o t)$$

$$x(t) = (a_1 - a_2)\cos(\omega_o t) \tag{8}$$

(a_1-a_2) represents the amplitude of SHMs. If both the amplitudes same, (i.e. a = a1 = a2) displacement of resultant wave is zero.

1.6. Superposition of two collinear harmonic oscillations with distinct frequency:

We discussed superposition of harmonic oscillations with varying angular frequencies in many situations. Different vibrations are applied to the human eardrums and the microphone diaphragm at the same time.

To make things easier, let's start by superimposing the two harmonic oscillations with same amplitude and minutely different frequencies, ω_1 and ω_2, where $\omega_1 \rangle \omega_2$

$$x_1(t) = a\cos(\omega_1 t + \varphi_1)$$

$$x_2(t) = a\cos(\omega_2 t + \varphi_2)$$

Respective phase difference (ϕ) between the two harmonic vibrations is:

$$\phi = (\omega_1 - \omega_2)t + (\varphi_1 - \varphi_2)$$

As first term is the time dependent so it is changes continuously with time and the second term (Φ_1 - Φ_2) is always constant with time. Which represent that, it doesn't play any important role here. Thus, we take zero initial phase of the two oscillations. Then, the displacement equation of two harmonic oscillations can be written as:

$x_1(t) = a\cos(\omega_1 t)$ and $x_2(t) = a\cos(\omega_2 t)$ (9)

As per Superposition principle resultant displacement is

$$x(t) = x_1(t) + x_2(t) = a\cos(\omega_1 t + \omega_2 t)$$

This equation can be rewritten as

$$x(t) = 2a\cos((\omega_1 - \omega_2)/2)t\cos((\omega_1 - \omega_2)/2)t$$

It shows the oscillatory motion with $(\omega_1 - \omega_2)/2$ angular frequency and having amplitude

$$2a\cos((\omega_1 - \omega_2)/2)t.$$

The average value of angular frequency is $\omega_{avg} = (\omega_1 + \omega_2)/2$ and the modulated angular frequency $\omega_{mod} = (\omega_1 - \omega_2)/2$ respectively.

We find that amplitude $a_{mod}(t) = 2a\cos\omega_{mod}(t)$ differs with the frequency $\omega_{mod}/2\pi = (\omega_1 - \omega_2)/4\pi$

This also means that the modulated amplitude requires values of ±2a, 0, in a single cycle or ω_{mod} $t = 0, \pi/2, , \pi, 3\pi/2$ and 2π respectively. Equation for resultant oscillations is

$x(t) = a_{mod}(t)\cos\omega_{avg}t$.

It is similar to displacement equation of SHM. However, this similarity is deceptive. The corresponding amplitude of modulated wave and its phase constant are:

$a_{mod}(t) = [a_1^2 + a_2^2 + 2a_1a_2\cos(2\omega_{mod}t)]^{1/2}$

$\theta_{mod} = [(a_1 - a_2)\sin\omega_{mod}t/(a_1 + a_2\cos\omega_{mod}t)]$

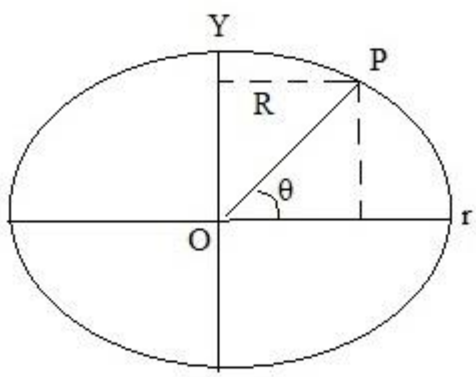
1.7. Superposition of many identical-frequency harmonic oscillations: Technique for Vector Addition:

This technique depends on the concept that, the uniform circular motion projected onto the circle's diameter is the displacement of the harmonic oscillation. It is crucial to relate the uniformly circular motion and SHM.

1.8. SHM with Uniform Circular Motion:

Consider a particle moving in a circular path with constant speed V. The radius vector which links the particle's position on the circumference to the circle's centre will rotate at a steady angular frequency.

At time t = 0, we consider direction of the radius vector is in x direction. The angle formed at any given time t, between radius vector and x axis is



$\boldsymbol{\theta}$ **= length of arc/radius of the circle = Vt/R**

At time t the position's x and y components are

$x = R\cos\theta \; and \; y = R\sin\theta$

Therefore, $\frac{dx}{dt} = -R\sin\theta\frac{d\theta}{dt}$

$= -\omega_0 R\sin\theta$

As $\frac{d\theta}{dt} = \omega_o = V/R$

In Similar way, we can also write

$$\frac{dy}{dt} = \omega_o R \cos\theta$$

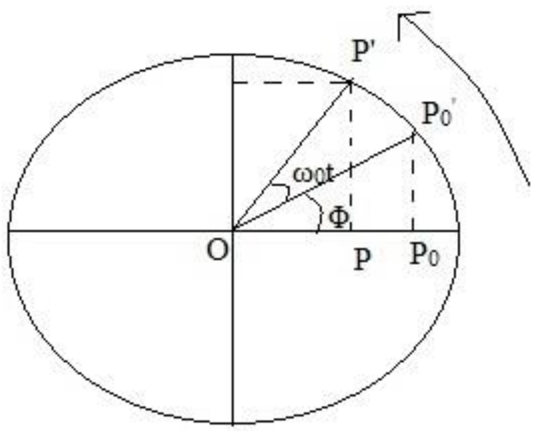
If we again differentiating with respect to time, we find that

$d_2x/dt^2 = -\omega_0{}^2x$ and $d^2y/dt^2 = -\omega_0 2y$

The aforementioned formula shows the differential equation, when a particle moves uniformly in a circle, SHM is carried out by its projections along the x and y axes. A simple 'harmonic motion may be viewed as the projection of uniformly rotating 'vector on reference axis.

Consider that vector OP' with |OP'| = a_0 is rotating with angular frequency (ω_0) in anticlockwise direction, see the Figure given below. Let P be foot of perpendicular drawn from P' on x axis. Then OP = x is projection of OP' on x axis. Point P performs basic harmonic motion along the x axis while vector OP' rotates at a steady pace.

Its period of oscillation is similar to the period of rotating vector OP'. Let OP_0' be initial position of the rotating vector and its projection OP_0 on x -axis is $a_0\cos\Phi$. If rotating vector moves from OP_0' to OP' in time t, then $\angle P'OP_0'$ $\omega_0 t$ and $\angle P'_O = (\omega_0 t + \Phi)$ Then we can write



$OP = OP'\cos\angle P'_O x$

or $x = a_0\cos(\omega_0 t + \phi)$

Therefore, point P executes simple harmonic motion along x-axis. If you project OP' on y -axis, you will find that point corresponding to the foot of the normal satisfies equation

$y = a_0\sin(\omega_0 t + \phi)$

This signifies that rotating vector can be resolved in two orthogonal components, and we can write

$r = x_x + y_y$

Where x_x and y_y are unit vectors along x and y axes, respectively.

1.9.Vector Addition:

Let us consider n harmonic oscillations superpose, all the harmonics having same amplitude and angular frequency which are a_0 and ω_0 respectively. Initial phases of successive oscillations differ by Φ_0. Let first of these oscillations be defined by equation

$x_1(t) = a_0\cos\omega_0 t$

Then, other oscillations are provided by:

$x_2(t) = a_0\cos(\omega_0 t + \Phi_0)$.....$x_n(t) = a_0\cos[\omega_0 t + (n-1)\Phi_0]$

From principle of superposition, resultant oscillation is expressed by

$x(t) = a_0[\cos\omega_0 t + \cos(\omega_0 t + \Phi) + \cos(\omega_0 t + 2\Phi_0) + ...\cos(\omega_0 t + (n-1)\Phi_0)]$

1.10. Oscillations in two dimensions*:*

Two-dimensional oscillatory motion is also feasible. The motion of a basic pendulum, whose bob is free to swing in any direction in the x-y plane, is the most famous example. The term "spherical pendulum" refers to this setup.

Pendulum is displaced in the x direction, and upon release, it receives an impulse in the y direction. While such a pendulum oscillates, the outcome is a composite motion, with maximal x-displacement occurring while y-displacement is low and y velocity is maximum, and vice versa. Recall that the frequency of the superposed SHMs will remain constant as the pendulum's time period is solely dependent on its acceleration due to gravity and the length of its cord. The curved path that results is typically an ellipse. Now, we apply the superposition principle to the situation in which two harmonic oscillations are orthogonal to one another.Two-dimensional oscillatory motion is also feasible. The motion of a basic pendulum with a free-swinging bob is the most famous example.

1.11. Superposition of Two Mutually Perpendicular Harmonic Oscillations of Same Frequency*:*

Assume that there are two oscillations that are mutually perpendicular, with amplitudes a_1 and a_2, angular frequency ω_0, $a_1 >> a_2$.

These are explained by equations

$x_1 = a_1\cos\omega_0 t$ and $x_2 = a_2\cos(\omega_0 t + \Phi)$

Here, we've assumed that the starting stages of vibrations along the x and y axes are, respectively, zero and Φ. In other words, Φ represents the phase difference between two vibrations. At first determine resultant oscillation for few particular values of phase difference:

Case 1: Φ = 0 or π

For $\Phi = 0$, $x = a_1\cos\omega_0 t$ and $y = a_2\cos\omega_0 t$

Therefore $\frac{y}{x} = \frac{a_2}{a_1}$ or $y = \frac{a_2}{a_1}x$

Likewise for $\Phi = \pi$ $x = a_1\cos\omega_0 t$ and $y = -a_2\cos\omega_0 t$

So that $y = -(a_2/a_1)x$

Straight lines traveling through the origin can be explained by the y, equation shown above. This indicates that the particle's subsequent motion is linear.

While the motion is along one diagonal when Φ = 0, it is along the opposite diagonal when Φ = π.

Case 2: Φ = π/2.

In this case two vibrations are provided by

$x = a_1\cos\omega_0 t$

$y = a_2\cos(\omega_0 t + \pi/2) = -a_2\sin\omega_0 t$

By squaring and adding resultant expressions, we get

$x^2/a_1^2 + y^2/a_2^2 = \cos2\Phi + \sin^2\Phi = 1$

This represents an equation of ellipse. As a result, the particle moves in an ellipse, with the x and y axes as its primary axis. a_1 and a_2 are semi-major and semi-minor axes of ellipse. As time increases x decreases from maximum positive value but y becomes more and more negative. Therefore, ellipse is explained in clockwise direction. If you analyze case when Φ = 3π/2 or Φ = π/2, you will get same ellipse, but motion will be in anticlockwise direction.

When amplitudes a_1 and a_2 are equal, $a_1 = a_2 = a$ Equation given above reduces to

$x^2 + y^2 = a^2$

This equation represents the circle of radius a. It signifies that ellipse reduces to circle.

Summary

In summary, the superposition of harmonic oscillations allows for the analysis of complex wave patterns by combining simpler oscillations. This principle is crucial in various fields, including acoustics, optics, and quantum mechanics, for understanding phenomena such as interference, resonance, and wave propagation.

Keywords

Collinear, Superposition, Beats, Interference

Objective Type Questions:

1. What does the Superposition Principle state regarding the behavior of linear systems?

   A) The output of a linear system is proportional to the input.

   B) The output of a linear system is the sum of the inputs.

   C) The output of a linear system is inversely proportional to the input.

   D) The output of a linear system is independent of the input.

2. When two collinear oscillations with equal frequencies are superimposed, what is the result?

   A) Constructive interference

   B) Destructive interference

   C) Beats

   D) Standing waves

3. What phenomenon occurs when two collinear oscillations with slightly different frequencies are superimposed?

   A) Resonance

   B) Diffraction

   C) Interference

   D) Beats

4. What characterizes the phenomenon of beats?

   A) Continuous interference pattern

   B) Rapid fluctuation in intensity or amplitude

   C) Stable interference pattern

   D) Maximum destructive interference

5. In beat phenomenon, the beat frequency is equal to:

   A) The sum of the frequencies of the two oscillations

   B) The difference between the frequencies of the two oscillations

   C) The product of the frequencies of the two oscillations

   D) The ratio of the frequencies of the two oscillations

Self Assessment

1. Define interference
2. Define diffraction
3. What is superposition?
4. Define linearity
5. What do you understand by the frequencies and time period

Chapter 2

Superposition of Two Perpendicular Harmonic Oscillations

Objectives

1. Understand how two perpendicular harmonic oscillations combine to form a resultant motion or wave pattern.
2. Explore graphical and analytical methods for representing and analyzing these combined oscillations.
3. Investigate Lissajous figures as visual representations of the superposition of perpendicular oscillations and their applications.

2.1.The superposition of two perpendicular harmonic oscillations by graphical and analytical methods is given below.

Given:

The number of perpendicular harmonic oscillators = 2.

To Find:

Using the graphical and analytical methods find the superposition of two perpendicular harmonic oscillation.

Solution:

Simple harmonic motion (SHM) is a type of periodic motion in which the amplitude of the object's displacement is directly proportionate to the restoring force acting on it. Which is responsible to bring the body in its equilibrium position?

Consider two perpendicular harmonic oscillators having same frequency. The SHM produced by the first oscillator in x-direction is given by,

$$x = A_1 \sin \omega t$$

The SHM produced by the second oscillator in y direction is given by,

$$y = A_2 \sin(\omega t + \delta)$$

Here δ defines the phase difference between the two perpendicular oscillators. The two oscillators' combined motion is a mixture of the two SHMs mentioned above.

The resultant motion of two oscillators follows a two-dimensional elliptical path. The equation for this path is obtained by eliminating the term "t" from x and y.

From the equation of SHM produced by the first oscillator, we get,

$$\sin \omega t = \frac{x}{A_1} \text{ or, } \cos\omega t = \sqrt{1-\left(\frac{x}{A_1}\right)^2}$$

Consider the SHM, $y = A_2 \sin(\omega t + \delta)$.

This can be written as,

$y = [\sin \omega t \times \cos \delta + \cos \omega t \times \sin \delta]$

Substituting the values of sin ωt and cosωt in the above equation, we get,

$$y = [\sin \omega t \times \cos \delta + \cos \omega t \times \sin \delta]$$

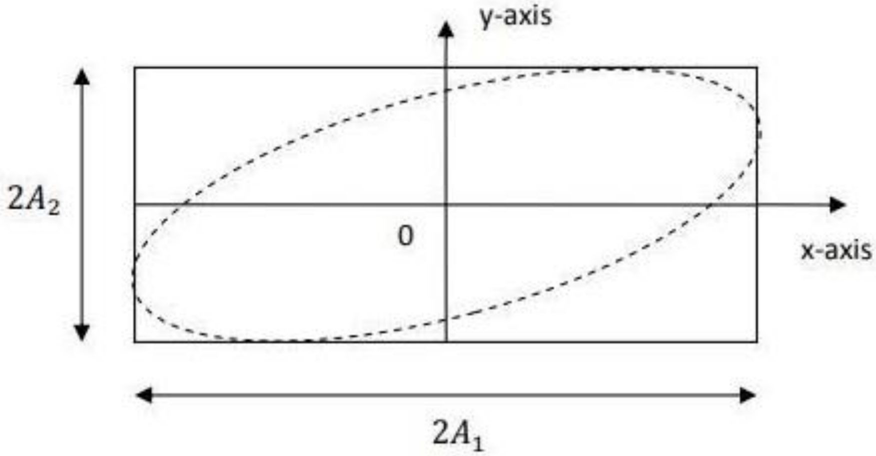
$$y = A_2\left[\frac{x}{A_1}\cos\delta + \sqrt{1-\left(\frac{x}{A_1}\right)^2}\,\sin\delta\right]$$

$$\text{Or } \left(\frac{y}{A_2} - \frac{x}{A_1}\cos\delta\right)^2 = \left(1-\left(\frac{x}{A_1}\right)^2\right)\sin^2\delta$$

$$\frac{x^2}{(A_1)^2} + \frac{y^2}{(A_2)^2} - \frac{2xy.\cos\delta}{A_1.A_2} = \sin^2\delta.$$

This is in the form of the equation for an ellipse. The motion of the two oscillators always remains inside the rectangle defined by $x = \pm A_1$ *and* $y = \pm A_2$.The graphical representation of the two-dimensional elliptical path followed by the oscillators is given below.

Therefore, the superposition of two perpendicular harmonic oscillations by graphical and analytical methods is represented as above.



2.2. Superposition of S.H.Ms (Lissajous Figures)

The resulting motion of two SHMs acting perpendicularly can take the shape of a straight line, circle, parabola, etc., it depends on the ration of frequencies of the two SHMs and the their initial phase difference. These figures are known as Lissajous figures.

Let the equations of two mutually perpendicular S.H.M's of same frequency be

$x = a_1 \sin\omega t$ and $y = a_2 \sin(\omega t+\phi)$

then the general equation of Lissajou's figure can be obtained as

x2a21+y2a22−2xya1a2cosϕ=sin2ϕ

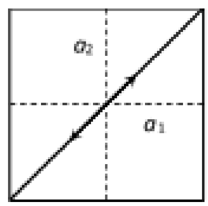
For

$\Phi = 0^o$

:

x2a21+y2a22−2xya1a2=0

(xa1−ya2)2=0

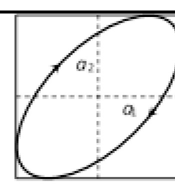
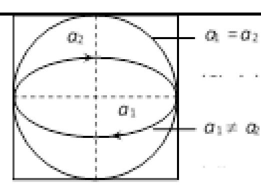


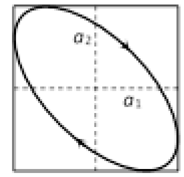
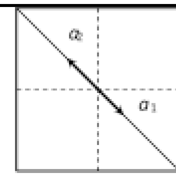
xa1=ya2⇒y=a2a1x

This is a straight line passes through origin and it's slope is

a2a1

2.3.Lissajou's figures in other conditions (with $\omega_1\omega_2=1$)

Phase diff. (φ)	**Equation**	**Figure**
π/4	x2a21+y2a22−2√xya1a2=12	Oblique ellipse 
π/2	x2a21+y2a22=1	

3π/4	x2a21+y2a22−2√xya1a2=12	Oblique ellipse 
π	xa1+ya2=0⇒ y=−a2a1x	Straight line 

For the frequency ratio

$\omega_1 : \omega_2 = 2:1$

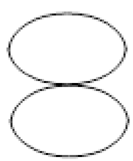
the two perpendicular S.H.M's are

$x=a_1\sin(\omega t+\phi)$

and

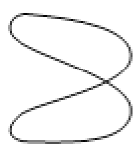
y=a2sinωt

Different Lissajou's figures as follows



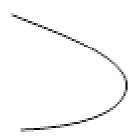
$\phi = 0, \pi, 2\pi$

Figure of eight



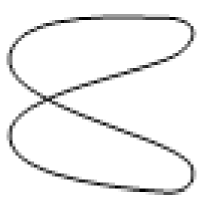
$\phi = \pi/4, 3\pi/4$

Double parabola



$\phi = \pi/2$

Parabola



$\phi = 5\pi/4, 7\pi/4$

Double parabola

$\phi = 3\pi/2$

Parabola

Summary

In summary, the superposition of two perpendicular harmonic oscillations can be analyzed using both graphical and analytical methods, with Lissajous figures providing a visual representation of their combined motion. These figures have practical applications in frequency measurement, instrument calibration, and educational demonstrations.

Keywords

Phase difference, Nodes and antinodes, standing waves, beat frequency

Objective type questions

1. What are Lissajous Figures?

   A) Geometric shapes formed by intersecting sine and cosine waves

   B) Patterns formed by intersecting circles

   C) Shapes formed by intersecting parabolas

   D) Figures created by intersecting straight lines

2. In a Lissajous Figure, if the frequencies of the two oscillating sources are not equal, what shape is produced?

   A) Circle

   B) Ellipse

   C) Parabola

   D) Straight line

3. Which parameter does not affect the shape of a Lissajous Figure?

   A) Amplitude

   B) Frequency

   C) Phase difference

   D) Velocity

4. When the phase difference between the two oscillating sources in a Lissajous Figure is $\pi/2$, what shape is produced?

   A) Circle

   B) Ellipse

C) Straight line

D) Square

5. In a Lissajous Figure, if the frequencies of the two oscillating sources are equal and the phase difference is zero, what shape is produced?

A) Circle

B) Ellipse

C) Parabola

D) Hyperbola

Self Assessment

1. What is oscillation
2. What do you mean by the oscillator
3. Examples of beats in daily life
4. Write down the applications of Lissajous Figures
5. Differentiate between the parallel and perpendicular oscillation

Chapter 3

Wave Motion

Objectives

1. Understand the characteristics and behavior of plane and spherical waves.
2. Differentiate between longitudinal and transverse waves and understand their properties.
3. Explore the concept of plane progressive (travelling) waves and their mathematical description.
4. Familiarize with the wave equation and its applications in describing wave propagation.
5. Understand particle and wave velocities in the context of wave motion.
6. Explore the differential equation governing wave motion.
7. Investigate the pressure associated with longitudinal waves.

Wave Motion is the motion of the waves. A disturbance resulting from energy propagating through space or a medium is called a wave. Wave motion is exemplified by the beams of light, the sound waves, and water ripples. We shall examine the various forms of waves found in nature as well as their motion in this post. Next, we will examine the characteristics and use of waves.

Further, we will also learn about sound waves. So let's start.

3.1. Functions of Waves

Wave Motion can perform the following functions.

1. Transfer Energy
2. Transfer Information
3. Cause disturbance in the media

Wave attributes are the distinctive characteristics shared by all waves. Wave motion is defined by these wave characteristics. Amplitude, frequency, wavelength, time period, phase, and phase difference are the parameters that constitute wave motion.

3.2. Speed of a Travelling Wave Motion

It is defined as: Wave Speed = distance covered time taken

In order to describe the phase at a place, following things are required

1. Displacement
2. Directional Velocity
3. Oscillation Number

Number of Dimensions a Wave Propagates Energy:

There are two and three dimensions types of waves are exist. A plane wave, for instance, is formed when the wave front or crest forms a line in two dimensions or a plane in three dimensions. There are also spherical waves in three dimensions and circular waves in two dimensions.

3.3. Periodic Wave Motion: When motion is repeated in equal intervals of time. Sine and cosine curves are the examples of such types of waves. A wave that repeats as a function of time and place is called a periodic wave, and its characteristics include amplitude, frequency, wavelength, speed, and energy. A periodic wave is connected to simple harmonic motion and repeats the same oscillation over multiple cycles, as in the case of the wave pool.

3.4.The Relationship between Path Difference and Phase Difference

It is defines as difference in the path traversed by the two waves, measured in terms of its wavelength. Path and phase difference both are related with each other. Nature of the interference pattern defines the phase difference. A phase difference is a quantum mechanical phenomenon.

Constructive interference occurs when path difference between two given waves is even multiple of $\lambda/2$ and destructive interference occurs if it is odd multiple of λ2.

3.5.Terminology of Waves

The terminologies used to describe a wave:

- **Amplitude (A):** The maximum value of displacement covered by a particle in the given medium is called Amplitude. Its SI unit is meter.
- **Time period (T):** The time required to complete one oscillation from its mean position is called time period (T). It is measured in seconds.
- **Wavelength (λ):** The distance between two successive crests or troughs for a wave is termed wavelength. Its S.I unit is a meter.
- **Frequency (n):** The number of oscillations performed by a particle in one sec is called frequency of waves. Its S.I unit is Hertz (Hz). Frequency is inversely proportional to time period i.e., $n=1/T$
- **Velocity (v):** The velocity of a wave is characterized as the distance it travel in a unit of time. The wave travels a distance equal to the wavelength (λ) during the period (T). Thus, the magnitude of the velocity of the wave is defined as,

$$Velocity\ of\ wave\ (v) = Frequency\ (n) \times Wavelength\ (\lambda)$$

- **Phase**: A particle's phase is the state in which it oscillates.

3.6. Classification of Wave Motion

Waves can be divided into three categories based on the way they move.

1. Mechanical Waves

Mechanical waves are defined as waves that can only travel through a material medium (require some medium for the propagation of wave motion).

Examples of Mechanical Wave Motion include waves in water, waves on a stretched string, seismic waves (earthquakes), sound waves, etc.

2. Electromagnetic Waves

Electromagnetic waves are those generated by oscillations in the electric and magnetic fields. There is no medium necessary for the wave motion of these waves to propagate.

Examples of electromagnetic waves are light waves (Photon)

3. Matter Waves

Matter waves are waves that are connected to the motion of protons, electrons, and other particles.

4. Standing Wave Motion

A unique kind of wave that oscillates inside a small area is called a stationary wave or standing wave. A standing wave's crest and trough remain stationary in space. In a standing wave, the oscillations at various places are in phase with one another. To put it briefly, this kind of wave motion is not space-propagating. Examples: Motion of Strings of a Sitar

3.7.Wave Speed of a Wave Motion on a Stretched String

The velocity of a standing wave is calculated by the mass per unit length of the string and the tension that exists in a stretched string.

V = TmL

3.8.Progressive Wave Motion

The waves which propagate in a media are called Progressive waves. The crest and trough of a progressive wave are stationary in space.

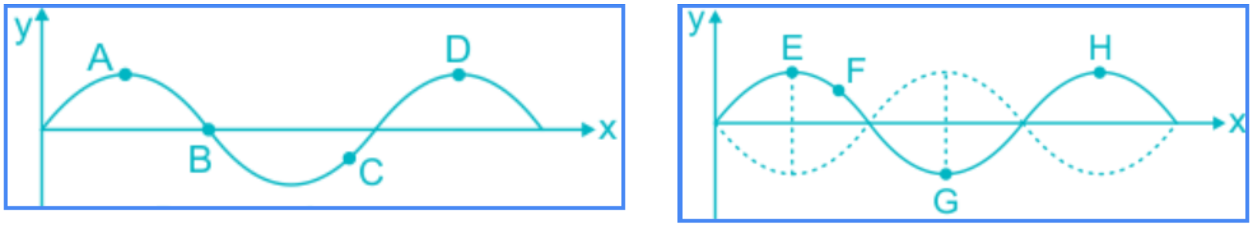


Figure 1 Standing and progressing waves as a displacement along the x and y axes, particle's phase shift also shown on various locations of the graph.

3.9. Classifications of Progressive Waves

The progressive waves can be classified into two types:

1. Longitudinal Wave
2. Transverse Wave

3.10. Longitudinal Wave

Waves in which a medium particle vibrates perpendicular to the direction in which the wave motion is propagating are known as longitudinal waves. Exp: Sound waves

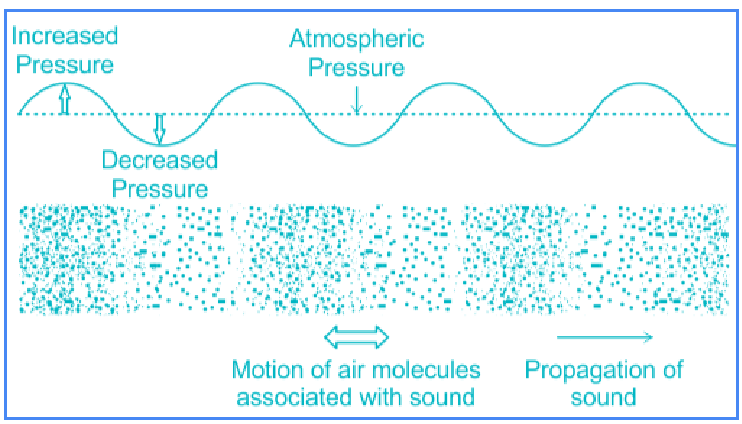


Figure 2

The air molecules in Figure 2 are represented by the dots. A region of compression or higher pressure is shown by more dots in that location, and a region of rarefaction or lower pressure is indicated by fewer dots in that same area.

Note: Because longitudinal waves propagate through continuously changing air densities, pressure and energy are transferred from one location to another during the process.

3.11. Transverse Wave Motion

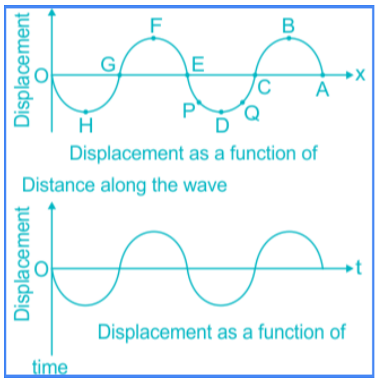
Waves in which the medium's particle vibrates perpendicular the direction in which the wave motion is propagating are known as transverse waves.

A transverse wave that travels over the water's surface as the water molecules vibrate perpendicular to it is called a water ripple.

Transverse waves propagate along a horizontal plane, but their particles vibrate up and down in a vertical direction.

Therefore, in a transverse wave motion, a particle rises from its mean location during a crest, and it falls below the mean position during a trough.

3.12. Wave Equation



Let us consider a wave that originates at origin O. The displacement of the wave motion at any given time t is given by

$y = a \sin t$

$y = a \sin \omega t$

A defines the amplitude of the wave.

A second wave starts behind the first wave. It lags by a difference of δ. The displacement of this wave at time t is given by,

y = a sin(t-)

$y = a \sin(\omega t - \delta)$

We know that path difference corresponds to a phase difference of 2π. Therefore, a path difference of 'x' will correspond to a phase difference of 2πxλ

Therefore, the displacement becomes,

$y = a \sin(\omega t - 2\pi\lambda)$

This is called the Wave Equation. It gives the position of any particle at any instant of time for a particular wave motion.

3.13. Energy and Power of a Wave Travelling Along a String

Consider a sinusoidal wave on a string created by a string vibrator to find the mathematical expression for a wave's energy, as seen in the following figure:

Potential Energy can be given by

$$U = \frac{1}{4}\mu\omega^2 a^2 \lambda$$

And Kinetic Energy is given by

$$KE = \frac{1}{4}\mu\omega^2 a^2 \lambda$$

And Total Energy is given by

$$T = U + KE = \frac{1}{2}\mu\omega^2 a^2 \lambda$$

3.14. Speed of Longitudinal Waves According to Newton's Formula

Now, in accordance with Newton's formula, the density and elasticity of the medium affect the longitudinal wave motion or pulse's velocity.

If the medium is uniform or unchanging, then the velocity of sound remains constant and it can be expressed as:

V = Elastic Property =$B\rho$

Here, B and ρ represent the bulk modulus and density of the medium.

Bulk modulus: A measure of a substance's resistance to compression, the bulk modulus is a constant.

It is defined as the ratio of any given material's volumetric strain to its volumetric stress.

For example:

Speed of sound depends upon bulk modulus of material and density of the material, as we know that

- Saltwater is about 2-4% denser than freshwater.
- But it also has a bulk modulus that's about 9% greater than that of freshwater.
- So overall the speed of sound in seawater is faster than tap water/freshwater.

Note:

For derivation, Newton assumes that the change in pressure and volume of a gas when sound waves are propagated through it is isothermal.

Whereas the above equation can also be modified as v=pρ−−√$v=p\rho$

Here, Bulk modulus (B) can be replaced by initial pressure due to the propagation of longitudinal waves.

Thus, we can see this is an ideal case which will not always be true in real life since the temperature of the medium may change or the system may not remain in an isothermal ideal state as assumed in Newton's formula.

Hence this was corrected by Laplace as shown below.

3.15. Speed of Longitudinal Waves (Sound) According to Laplace's Correction

Using Newton's assumption Laplace pointed out that it will not give correct results as it is considered for an ideal condition.

Laplace states that the pressure-volume changes that take place in a gas during the passage of a sound wave must be adiabatic in character rather than isothermal.

He assumed that there is no heat exchange taking place as the sound propagates through air.

Hence according to Laplace's correction formula for the speed of sound in a gas is

v=$\sqrt{\gamma p}$=√$v=\gamma p\rho$

3.16. Factors Affecting the Speed of Wave Motion:

Elasticity:

The tendency of a material to hold its shape and not deform in response to an applied force is referred to as its elastic property. Since the atoms in elastic materials are strongly connected, the wave motion speed is higher in these materials.

Density:

The mass per unit volume is defines the density of the substance. The material will transmit waves more slowly if it is denser due to larger molecules.

Temperature:

In a warm environment, the wave motion moves more quickly than in a cold one. The characteristics of the medium include its density and elasticity.

Humidity:

In a humid environment, the wave motion travels faster as compared to the dry environment.

The direction of wind:

The direction of the wind has a significant impact on the wave's motion speed. For instance, a wave moving in the direction of the wind will travel faster; conversely, a wave moving against the wind will move slower.

Summary: In summary, understanding the behavior of waves involves grasping concepts such as wave types (plane and spherical), wave equations, particle and wave velocities, and the pressure associated with longitudinal waves. These concepts are fundamental in various fields, including physics, acoustics, and seismology.

Keywords

Wave nature, Wave motion in 1D, 2D, 3D, Matter waves

Objective Type Questions:

1. Electromagnetic waves are considered to be which of the following types?

   a) Transverse

   b) Longitudinal=

   c) Both Transverse & Longitudinal

   d) Neither longitudinal nor transverse

2. Sound travels through which medium?

a. Solid
b. Liquid
c. Gas
d. All the above

3. Sound waves in air is an example of ——————

a. Longitudinal wave
b. Transverse wave
c. Electromagnetic wave
d. None of the options

4. The waves, in which the particles of the medium vibrate in a direction perpendicular to the direction of wave motion, is known as ___________

   a) Transverse waves

   b) Longitudinal waves

c) Propagated waves

d) Magnetic waves

5. For a wave propagating in a medium, identify the property that is independent of the others.

a) Velocity

b) Wavelengths

c) Frequency

d) All these depend on each other

Self Assessment

1. Examples of waves in daily life
2. What is waves ?
3. Write down the examples of transverse waves
4. Write down the wave equation in 1 D, 2 D, 3D
5. What is particle velocity?

Chapter 4
Velocity of Waves

Objectives

1. Understand the factors influencing the velocity of transverse vibrations in stretched strings.
2. Explore the factors affecting the velocity of longitudinal waves in fluids within pipes.
3. Examine Newton's formula for the velocity of sound and its derivation.
4. Understand Laplace's correction and its role in refining Newton's formula for the velocity of sound.

4.1.Velocity of a Transverse Waves in a Stretched String

Let's calculate the velocity of transverse wave travelling on a string. We will calculate the wave's velocity in two scenarios:

1. The Transverse wave's speed along a stretched string.
2. The Longitudinal wave velocity in an elastic material.

4.2.Velocity of Transverse Wave Along a Stretched String

Let's calculate the transverse travelling wave velocity on a string. As seen in Figure 4.1(a), the wave pulses move towards the right end of the rope with a velocity (v) when a jerk is applied at one end (the left end).

This indicates that, as regard to an observer in a rest frame, the pulses travel with velocity (v). Assuming that an observer goes in the same direction as the wave pulse travelling and with the same velocity v. Observer note that the rope is travelling at the same velocity v as the pulse, while the wave pulse remains stationary.

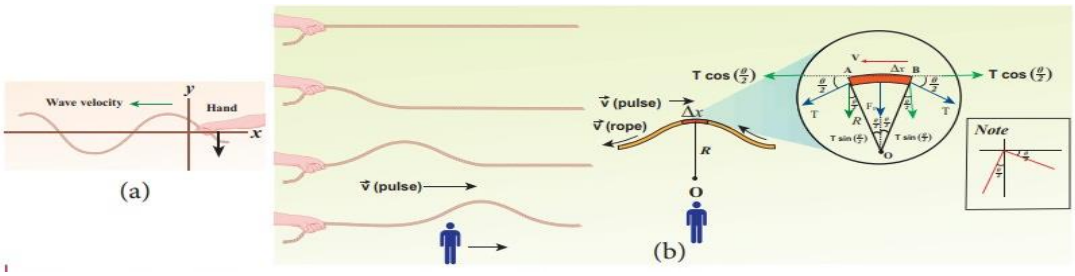


Figure 4.1 (a) Transverse wave in a stretched string (b) infinitesimal segment of width (dx) in a stretched string is zoomed and the pulse seen from an observer frame who moves with velocity v.

Consider an infinitesimal segment of width □x in the string, as shown in the Figure 4.1 (b). Let A and B be two points on the string at an instant of time. The length and mass of the segment is *dl* and *dm* respectively. Now from the definition of linear mass density (μ):

$$\mu = \frac{dm}{dl} \tag{1}$$

$$dm = \mu dl \tag{2}$$

The segment AB represent an arc of a circle whose centre is at O. Radius of the circle is R, arc subtending an angle θ at the origin O as shown in Figure 4.1 (b). The angle θ can be expressed as θ = dl/R when considering arc length and radius. The string's tension provides a centripetal acceleration that is

$$a_{cp} = \frac{v^2}{R} \tag{3}$$

If mass of the segment is incorporated then centripetal force can be calculated as

$$F_{cp} = \frac{(dm)v^2}{R} \tag{4}$$

By the simplification and after putting the value of dm from equation (2) in equation (4)

$$F_{cp} = \frac{\mu v^2 dl}{R} \tag{5}$$

At points A and B, a tension (T) acts along the tangent of the string's elemental segment. It is possible to disregard variations in the tension force because of the extremely narrow arc length. T can be split into two components: a vertical component, T sin(θ/2) and a horizontal component, T cos(θ/2).

Magnitude of horizontal component at A and B end are equal but direction is opposite, therefore, cancel each other. Length of the arc AB is very small the vertical components act towards the centre of the arc. As direction is same, hence, they add up. The net radial force F_r is

$$F_r = 2T\sin\left(\frac{\theta}{2}\right) \tag{6}$$

As amplitude is very small, when it is compared with the length of the string, the sine of small angle is approximated as sin (θ/2) ≈ θ/2. Hence, equation (6) can be written as

$$F_r = 2T\left(\frac{\theta}{2}\right) = T\theta \tag{7}$$

Put the value of θ = dI/R, in equation number (7):

$$F_r = T\frac{dl}{R} \qquad (8)$$

After applying the Newton's second law of motion the string segment in the radial direction, in the equilibrium condition, both the radial component of the force is equal to the centripetal force. After equating equation (5) and (8).

$$T\frac{dl}{R} = \frac{\mu v^2 dl}{R}$$

$$v = \sqrt{\frac{T}{\mu}} \qquad (9)$$

After the whole derivation following points are concluded:

The velocity of the wave in string is (1) square root of the tension force and (2) inversely proportional to the linear mass density. (3)It is independent of shape of the wave.

4.3.Velocity of Longitudinal Waves in a Fluid Propagating in Pipe

Imagine an elastic medium (air) with a fixed mass that is kept under pressure P inside a long tube (cylinder) with a cross sectional area of A. By retaining a vibrating tuning fork at one end of the tube or by displacing the fluid with a piston, one can create longitudinal waves in the fluid. Assume for the moment that the cylinder's axis and the wave's direction of propagation coincide.

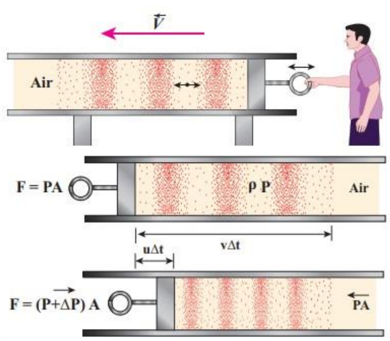


Figure 4.2 Longitudinal waves in the fluid, by displacing the fluid using a piston.

Let ρ be the fluid's initial density at rest. The piston at the left end of the tube is started moving toward the right at t = 0 and has a speed of u.

Let v be the elastic wave's velocity and u be the piston's velocity. The piston's displacement in time interval Δt is given by Δd = uΔt. Hence, Δx = vΔt is the distance travelled by the elastic disturbance. Let m represent the mass of the air that has travelled at a speed of v in t seconds.

Therefore,

$$m = \rho A\Delta x = \rho A(v\Delta t)$$

Subsequently, the momentum generated by the piston moving at velocity u is

$$p = [\rho A(v\Delta t)]u$$

From the definition of impulse, i.e. change in the momentum

The net impulse is

$$I = (\Delta PA)\Delta t$$

$$or\ (\Delta PA)\Delta t = [\rho A(v\Delta t)]u$$

$$\Delta P = \rho vu \tag{10}$$

The tiny volume element (ΔV) of the air experiences regular compressions and rarefactions when the sound wave travels through it. Thus, another way to express the pressure change is as

$$\Delta P = B\frac{\Delta V}{V}$$

where, V and B are original volume and bulk modulus of the elastic medium.

But $V = A\Delta x = Av\Delta t$ and $\Delta V = A\Delta d = Au\Delta t$

Therefore,

$$\Delta P = B\frac{Au\Delta t}{Av\Delta t} = B\frac{u}{v} \tag{11}$$

Comparing equation (11) and equation (10), we get

$$\rho vu = B\frac{u}{v}\ or\ v^2 = \frac{B}{\rho}$$

$$v = \sqrt{\frac{B}{\rho}} \tag{12}$$

A longitudinal wave in an elastic medium has a general velocity of $v = \sqrt{E/P}$, where E is the medium's modulus of elasticity.

Case 1 : Solid :

1) One dimension rod (1D)

$$v = \sqrt{\frac{Y}{\rho}} \tag{13}$$

Where ρ is the rod's density and Y indicates the material's Young's modulus. Only Young's modulus will be present in the 1D rod.

2) Three dimension rod (3D)

The speed of longitudinal wave in a solid is

$$v = \sqrt{\frac{K + \frac{4}{3}\eta}{\rho}} \quad (14)$$

Where η, K and ρ are modulus of rigidity, bulk modulus and density of the rod.

Case 2 : Liquids:

$$v = \sqrt{\frac{K}{\rho}} \quad (15)$$

4.4.Propagation of Sound Waves

By using Newton's methods we can calculate the speed of sound in air. We also go over the Laplace correction and the variables that affect sound in the air.

Sound waves are longitudinal in nature therefore compressions and rarefactions occur throughout their propagation. The speed of sound in the air is calculated using Newton's method in the part that follows. We also go over the Laplace correction and the variables that affect sound in the air.

4.5.Speed of Sound Waves in Air (Newton's Formula)

Sir Isaac Newton postulated that the production of compression and rarefaction occurs very slowly during sound propagation in air, that suggesting that the process is isothermal in nature. In other words, the heat generated during compression—when pressure rises and volume falls—and the heat lost during rarefaction—when pressure falls and volume rises—occur throughout time, by keeping the medium's temperature constant.

Thus, by converting the air molecules into an ideal gas, Boyle's law governs the changes in pressure and volume mathematically.

PV = Constant (16)

On differentiating the equation (16),

$$PdV + VdP = 0$$

$$P = -V\frac{dP}{dV} = B_T \quad (17)$$

where, B_T is an bulk modulus at constant temperature. Substituting equation (17) in equation (16), the speed of sound in air is

$$V_T = \sqrt{\frac{B_T}{\rho}} = \sqrt{\frac{P}{\rho}} \quad (18)$$

Since *P* is the pressure, at NTP it is 76 cm of mercury. we have

$$P = (0.76 \times 13.6 \times 10^3 \times 9.8)\ \text{N m}^{-2}$$

$$\rho = 1.293\ \text{kg m}^{-3}$$

Then the speed of sound in air at NTP is:

$$v_T = \sqrt{\frac{(0.76 \times 13.6 \times 10^3 \times 9.8)}{1.293}}$$

$$= 279.80\ \text{m s}^{-1} \approx 280\ \text{ms}^{-1}\ \textit{(theoretical value)}$$

However, experimental observations of the speed of sound in air at 0°C show that it is 332 m s-1, which is over 16% higher than the theoretical number. This error cannot be ignored.

4.6. Laplace's Correction

By assuming that the particles oscillate quickly during sound propagation through a medium, compression and rarefaction happen quickly; Laplace satisfactorily corrected this discrepancy in 1816. Because air (the medium) is a poor heat conductor, neither the compression-induced heat exchange nor the rarefaction-induced cooling effect occurs. Sound propagation is an adiabatic process since temperature is no longer regarded as a constant in this context. The gas obeys Poisson's law by adiabatic considerations, not Boyle's law as Newton believed.

$$PV^{\gamma} = \text{Constant} \qquad (19)$$

Where the ratio between specific heat at constant pressure (C_P) and at constant volume (C_v).

Differentiating equation (19) on both sides, we get

$$V^{\gamma} dP + P\left(\gamma V^{\gamma-1} dV\right) = 0$$

$$\gamma P = -V\frac{dP}{dV} = B_A \qquad (20)$$

where, Bulk modulus (B_A) is the bulk modulus of air at constant heat. Now, substituting equation (20) in (18), the speed of sound in air is

$$V^{\gamma} = \sqrt{\frac{B_A}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\gamma}\,V_T \qquad (21)$$

Since the majority of the gases in air are diatomic (such as nitrogen, oxygen, hydrogen, etc.), we use γ = 1.47.

As a result, the speed of sound in air is $V_A = \sqrt{1.4} * 280\, m\, s^{-1} = 331.30\, ms^{-1}$ which is very much closer to the experimental results.

4.7. Speed of Sound's Depends on Various Factors

Consider the case of an ideal gas with the equation of state

$$PV = nRT \qquad (22)$$

Where P stands for pressure, V for volume, T for temperature, and R is the universal gas constant. The number of moles is represented by n.

One way to express equation (22) for a given mass of a molecule is as

$$\frac{PV}{T} = Consntat \qquad (23)$$

For m fixed mass, density of the gas and its volume are is inversely related i.e.,

$$\rho \propto \frac{1}{V}, V = \frac{m}{\rho} \qquad (24)$$

Substituting equation (23) in equation (24), we get

$$\frac{P}{\rho} = cT \qquad (25)$$

where *c* is constant.

The speed of sound in air given in equation (22) can be written as

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\gamma cT} \qquad (26)$$

From the relationship above, we can see that

(a) Effect of Pressure (P) :

When pressure changes at a fixed temperature, density changes in proportion, resulting in a constant ratio (P/ρ).

It shows that, at a constant temperature, the speed of sound is does not dependent on pressure. The speed of sound will not change at the top or bottom of a mountain if the temperature is constant at both locations. However, in reality, a mountain's summit and bottom have different temperatures, which cause the sound to travel at different speeds.

(b) Effect of Temperature (T):

Since $v \propto \sqrt{T}$,

Speed of sound varies directly to the square root of temperature in Kelvin (K).

Let v_0 and v be the velocity of sound wave at 0° C or 273 K temperature and at any arbitrary temperature T^oK, then

$$\frac{v}{v_0} = \sqrt{\frac{T}{273}} = \sqrt{\frac{273+t}{273}}$$

$$v = v_0\sqrt{1+\frac{t}{273}} \cong v_0\left(1+\frac{t}{546}\right)$$

(using binomial expansion)

Since $v_0 = 331ms^{-1}$ at 0^0C, v is velocity at any temperature in t^0C.

$$v = (331 + 0.60t)\, ms^{-1}$$

The velocity of sound in air thus rises by 0.61 m s-1 with one degree Celsius increasing the temperature rise. Keep in mind that as temperature rises, molecules vibrate more quickly due to a gain in thermal energy, increasing sound speed in the process.

(c) Effect of Density

Let's examine two gasses that have the same temperature and pressure but differing densities. Then, the two gases' respective sound velocities are

$$v_1 = \sqrt{\frac{\gamma_1 P}{\rho_1}} \qquad (27)$$

and

$$v_2 = \sqrt{\frac{\gamma_2 P}{\rho_2}} \qquad (28)$$

By dividing the equation (27) and equation (28), we get

$$\frac{v_1}{v_2} = \frac{\sqrt{\frac{\gamma_1 P}{\rho_1}}}{\sqrt{\frac{\gamma_2 P}{\rho_2}}} = \sqrt{\frac{\gamma_1 \rho_2}{\gamma_2 \rho_1}}$$

As gases having same values of

$$\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}} \qquad (29)$$

Therefore, the square root of the and sound's velocity and its velocity are inversely related with each other.

(d) Effect of Moisture (dryness)

As density of dry air is 0.625 times lesser then the moist air, an increase in humidity or the presence of moisture in the air causes a drop in density. When a result, sound travels faster when humidity rises. Using equation (26)

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

Here ρ_1, v_1, and ρ_2, v_2 is the density of sound and its speed in dry and moist air.

$$\frac{v_1}{v_2} = \frac{\sqrt{\frac{\gamma_1 P}{\rho_1}}}{\sqrt{\frac{\gamma_2 P}{\rho_2}}} = \sqrt{\frac{\rho_2}{\rho_1}} \quad \text{if } \gamma_1 = \gamma_2$$

P represents the overall atmospheric pressure, hence it can be demonstrated that

$$\frac{\rho_2}{\rho_1} = \frac{P}{p_1 + 0.625 p_2} \qquad (30)$$

Where the partial pressures of water vapour and dry air are denoted by p_1 and p_2, respectively.

(e) Effect of Wind

Wind is also responsible to change the sound's speed. Sound speed increases where the wind is blowing while in the converse route, it decreases.

Summary

In summary, understanding the velocity of transverse vibrations in stretched strings and longitudinal waves in fluids, as well as Newton's formula for the velocity of sound and Laplace's correction, provides insight into the behavior of waves in different media and helps in accurately predicting wave velocities in various contexts.

Keywords

Transverse nature, Wave behavior, Newton’s formula, Sound waves

Objective Type Questions:

1. What is the velocity of transverse vibrations in a stretched string proportional to?
   A) The tension in the string and the square root of its linear density
   B) The length of the string and the square of its tension
   C) The amplitude of the vibrations and the density of the string
   D) The frequency of the vibrations and the diameter of the string

2. Which of the following factors affects the velocity of transverse vibrations in a stretched string?
   A) Temperature
   B) Mass of the string
   C) Cross-sectional area of the string
   D) Material of the string

3. The velocity of longitudinal waves in a fluid in a pipe depends primarily on:
   A) The density of the fluid and the length of the pipe
   B) The temperature of the fluid and the diameter of the pipe
   C) The bulk modulus of the fluid and the density of the fluid
   D) The pressure of the fluid and the material of the pipe

4. Newton’s formula for the velocity of sound relates the speed of sound in a gas to which properties of the gas?
   A) Pressure and volume
   B) Temperature and density
   C) Mass and volume
   D) Temperature and pressure

5. Laplace’s correction to Newton’s formula for the velocity of sound accounts for the influence of which factor?

A) Humidity

B) Altitude

C) Pressure

D) Temperature

Self Assessment

1. What is Transverse Vibrations?
2. Define Velocity
3. Define Fluid
4. What is the velocity of sound in air?
5. What is the Laplace’s Correction.?

Chapter 5

Superposition of Two Harmonic Waves

Objectives

1. Understand the principle of superposition and its application to the combination of two harmonic waves.
2. Explore the formation and properties of standing (stationary) waves in a string with fixed and free ends.
3. Analyze the behavior of phase and group velocities.
4. Determine the energy of vibrating string.

5.1. Standing Waves or Stationary Wave

The wave returns to the original medium after colliding with the stiff boundary, where it may cause interference with the preceding waves.

5.2. Standing Waves

Explanation

The wave returns to the original medium after colliding with the stiff boundary, where it may cause interference with the preceding waves. We get a pattern, which are standing or stationary waves. Think about two harmonic progressive waves that flow in opposite directions but having the equal amplitude, velocity (created by strings). The displacement of the first incident wave is

$$y_1 = A\sin(kx - \omega t)$$
(waves move toward right) (1)

An the second wave's (the reflected wave) displacement is

$$y_2 = A\sin(kx + \omega t)$$
(waves move toward left) (2)

Due to the superposition principle, both will interact with one another; the displacement after superposition is

$$y = y_1 + y_2 \tag{3}$$

After substitution of equation (1) and (2) in equation (3)

$$y = A\sin(kx - \omega t) + A\sin(kx + \omega t) \quad (4)$$

Using trigonometric relations, we (4) becomes

$$y(x, t) = 2A\cos(\omega t)\sin(kx) \quad (5)$$

This image depicts a standing or stationary wave, which denotes that it is not moving forward or backward in contrast to progressive or travelling waves.

Moreover, there is a more compact way to express the particle displacement in equation (5):

$y(x,t) = A' \cos(\omega t)$ (6)

When the expression A' = 2Asin(kx) indicates the amplitude of specific string element which is oscillating SHM. This amplitude's maximum happens at locations for which

$\sin(kx) = 1$

$$\sin(kx) = 1 \Rightarrow kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots = m\pi$$

Where m accepts values that are half integral or half integer. The term "antinode" refers to the location of highest amplitude. The anti-nodal positions can be expressed as wave number in terms of wavelength by using the formula

$$x_m = \left(\frac{2m+1}{2}\right)\frac{\lambda}{2}, \text{ where, } m = 0,1,2\ldots$$

For $m = 0$ we have maximum at

$$x_0 = \frac{\lambda}{2}$$

For $m = 1$ we have maximum at

$$x_1 = \frac{3\lambda}{4}$$

For $m = 2$ we have maximum at

$$x_2 = \frac{5\lambda}{4}$$

and so on.

It is possible to calculate the separation between two consecutive anti-nodes using

$$x_m - x_{m-1} = \left(\frac{2m+1}{2}\right)\frac{\lambda}{2} - \left(\frac{(2m+1)+1}{2}\right)\frac{\lambda}{2} = \frac{\lambda}{2}$$

In a similar way, the space contains certain sites where the amplitude A' is minimum. These points can be found by setting

$$\sin(kx) = 0 \Rightarrow kx = 0, \pi, 2\pi, 3\pi, \ldots = n\pi$$

Where in values taken by n are integer. It should be noted that these locations are known as nodes. The placements of nth nodes are provided by,

$$x_n = n\frac{\lambda}{2} \text{ where, n = 0,1,2,...}$$

At different values of n like = 0, 1 we get minimum which is $x_0 = 0$, and $x_1 = \lambda/2$ and maximum at n = 2 it is $x_2 = \lambda$ and so on.

Any two consecutive nodes' distance can be computed as

$$x_n - x_{n-1} = n\frac{\lambda}{2} - (n-1)\frac{\lambda}{2} = \frac{\lambda}{2}.$$

EXAMPLE

Calculate the separation between anti-nodes its neighborhood node.

Solution

For *nth* mode, the separation between anti-node and its neighborhood node is

$$\Delta x_n = \left(\frac{2n+1}{2}\right)\frac{\lambda}{2} - n\frac{\lambda}{2} = \frac{\lambda}{4}$$

5.3.Properties of Stationary Waves

(1) When a wave disturbance is contained within two inflexible boundaries, it is said to be stationary. It follows that the wave stays neither stationary at its location, and neither moves forward nor backward in a medium. As a result, they are referred to as "stationary waves or standing waves."

(2) There are places in the wave's region where the amplitude is at its highest, known as anti-nodes, and places where it is at its lowest, or zero, known as nodes.

(3) Separation is $\lambda/2$, between two successive nodes, or anti-nodes.

(4) Separation is λ/4, between node and its adjacent anti-node.

(5) There is no energy transfer along the standing wave.

5.4.Difference Between Progressive Waves and Standing Waves

S.No.	Progressive waves	Stationary waves
1.	Crests and troughs are formed in transverse progressive waves, and compression and rarefaction are formed in longitudinal progressive waves. These waves move forward or backward in a medium i.e., they will advance in a medium with a definite velocity.	Crests and troughs are formed in transverse stationary waves, and compression and rarefaction are formed in longitudinal stationary waves. These waves neither move forward nor backward in a medium i.e., they will not advance in a medium.
2.	All the particles in the medium vibrate such that the amplitude of the vibration for all particles is same.	Except at nodes, all other particles of the medium vibrate such that amplitude of vibration is different for different particles. The amplitude is minimum or zero at nodes and maximum at anti-nodes.
3.	These wave carry energy while propagating.	These waves do not transport energy.

5.5. Stationary Waves in "Sonometer"

Sono means "related to sound," and sonometer denotes measures connected to sound. It is a tool used to show how the tension, length, and mass of a string relate to the frequency of sound generated in the transverse standing wave within the string. As a result, we may calculate the following quantities with this device:

1. Frequency of AC current or frequency of tuning fork
2. String's tension (T)
3. Hanging mass (m)

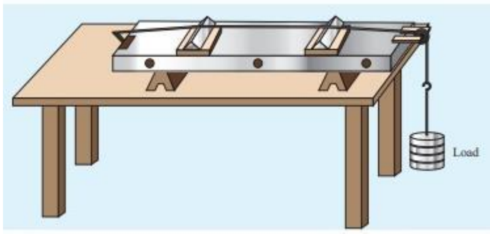


Figure: 5.1 Sonometer

5.6. Construction

It sonometer consists of a one-meter-long empty box with a thin, homogeneous metallic string attached. As seen in Figure 5.1, a hook is attached at one of its end, while the other end is attached to a weight hanging via a pulley. Monochord is another name for it because it only uses one string. To raise the tension in the wire, weights are attached to the free end. To alter the vibrating length of the stretched wire, two wooden knives with changeable positions are placed above the board.

Working

As a result of the formation of a transverse stationary or standing wave, nodes form at the knife edges P and Q. Anti-nodes form in the spaces between the knife edges. If the vibrating element's length is l, then

$$l = \frac{\lambda}{2} \Rightarrow \lambda = 2l$$

Let f, T and μ be the frequency in the segment, string tension and its mass density using the following equation:

$$f = \frac{v}{\lambda} = \frac{1}{2l}\sqrt{\frac{T}{\mu}} \text{ in Hertz}$$

Let d and ρ be the diameter and density of the string's material. Next, mass per unit length (μ),

$$\mu = \text{Area} \times \text{density} = \pi r^2 \rho = \frac{\pi \rho d^2}{4}$$

$$\text{frequency } f = \frac{v}{\lambda} = \frac{1}{2l}\sqrt{\frac{T}{\frac{\pi d^2 \rho}{4}}}$$

$$\therefore \quad f = \frac{1}{ld}\sqrt{\frac{T}{\pi \rho}} \qquad (:$$

Example

Let Fundamental frequency of the string be f. l_1, l_2 and l_3 are the three segments of the string so that f_1, f_2 and f_3 are the fundamental frequencies of each segment. Show that

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

Solution

For a T-fixed tension and μ-mass density, frequency and string length are inversely proportional to each other i.e.

$$f \propto \frac{1}{l} \Rightarrow f = \frac{v}{2l} \Rightarrow l = \frac{v}{2f}$$

For the first length segment

$$f_1 = \frac{v}{2l_1} \Rightarrow l_1 = \frac{v}{2f_1}$$

For the second length segment

$$f_2 = \frac{v}{2l_2} \Rightarrow l_2 = \frac{v}{2f_2}$$

For the third length segment

$$f_3 = \frac{v}{2l_3} \Rightarrow l_3 = \frac{v}{2f_3}$$

Therefore, the total length

$$l = l_1 + l_2 + l_3$$

$$\frac{v}{2f} = \frac{v}{2f_1} + \frac{v}{2f_2} + \frac{v}{2f} \Rightarrow \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

5.7. Fundamental Frequency and Overtones

Now, let's maintain the strict boundaries at x = 0 and x = L and use the string's wiggle motion to create a standing wave (much like when you pluck guitar strings). A certain wavelength of standing waves is generated. As amplitude disappear at the limits, the below mentioned requirement are satisfy at the boundary.

$$y(x = 0, t) = 0 \text{ and } y(x = L, t) = 0 \qquad (6)$$

Given that the nodes that have formed are separated by λn/2, we have n(λn/2) = L, where λn is the particular wavelength that satisfies the given boundary constraints, L is the length between the two borders, and n is an integer. Therefore,

$$\lambda_n = \left(\frac{2L}{n}\right)$$

As a result, not every wavelength is permitted. The permitted wavelengths ought to align with the designated boundary conditions; that is, in the case of n = 1, the first vibration mode possesses a particular wavelength of λl = 2L. In a similar vein, the second mode of vibration for n = 2 has a certain wavelength.

$$\lambda_2 = \left(\frac{2L}{2}\right) = L$$

At third mode n = 3, vibration has specific wavelength

$$\lambda_3 = \left(\frac{2L}{3}\right)$$

and so on.

It is possible to compute the natural frequency, i.e. frequency of each mode of vibration.

$$f_n = \frac{v}{\lambda_n} = n\left(\frac{v}{2L}\right)$$

In nature, the fundamental frequency is the lowest frequency that exists.

$$f_1 = \frac{v}{\lambda_1} = \left(\frac{v}{2L}\right)$$

It is first over tone - second natural frequency.

$$f_2 = 2\left(\frac{v}{2L}\right) = \frac{1}{L}\sqrt{\frac{T}{\mu}}$$

It is *second over tone* - third natural frequency.

$$f_3 = 3\left(\frac{v}{2L}\right) = 3\left(\frac{1}{2L}\sqrt{\frac{T}{\mu}}\right)$$

Consequently, one can calculate the nth natural frequency as an integral multiple of the fundamental frequency, that is to say,

$f_n = nf_1$, where n is an integer (7)

Natural frequencies are referred to as harmonics if they are expressed as an integral multiple of fundamental frequencies. The fundamental frequency is referred to as the first harmonic, which is $f_1 = f_1$. The second harmonic is $f_2 = 2f_1$, the third harmonic is $f_3 = 3f_1$, and so on.

EXAMPLE

Imagine that you are plucking an 80 cm-long guitar string that weighs 0.32 g and has a tension of 80 N. After it is plucked, calculate the first four lowest frequencies that are produced.

Ans.

The velocity of the wave

$$v = \sqrt{\frac{T}{\mu}}$$

The string's length, $L = 80\ cm = 0.8\ m$ the string's mass, $m = 0.32\ g = 0.32 \times 10^{-3}\ kg$

Thus, the linear density of mass,

$$\mu = \frac{0.32 \times 10^{-3}}{0.8} = 0.4 \times 10^{-3}\ kg\, m^{-1}$$

$$T = 80\ N$$

$$v = \sqrt{\frac{80}{0.4 \times 10^{-3}}} = 447.2\ \text{m s}^{-1}$$

The wavelength corresponds the fundamental frequency f_1 is

$$\lambda_1 = 2L = 2 \times 0.8 = 1.6\ m$$

The wavelength λ_1 corresponds to the fundamental frequency f_1.

$$f_1 = \frac{v}{\lambda_1} = \frac{447.2}{1.6} = 279.5\,Hz$$

Similarly, the second, third, and fourth harmonics' respective frequencies are

$$f_2 = 2f_1 = 559\ Hz$$
$$f_3 = 3f_1 = 838.5\ Hz$$
$$f_4 = 4f_1 = 1118\ Hz$$

law of transverse vibrations in the stretched strings

The transverse vibrations of stretched strings obey three laws, those are listed below:

(i) The law of length:

The frequency of a given wire with fixed mass per unit length μ and fixed tension T varies inversely with the vibrating length. Consequently,

$$f \propto \frac{1}{l} \Rightarrow f = \frac{C}{l}$$

$\Rightarrow l \times f = C$, it is a constant

(ii) The law of tension:

The frequency swings in direct proportion to the square root of the tension (T) when the mass per unit length (μ) and vibrating length (l) are fixed.

$$f \propto \sqrt{T}$$

$$\Rightarrow f = A\sqrt{T}, \text{ where A is a constant}$$

(iii) The law of mass:

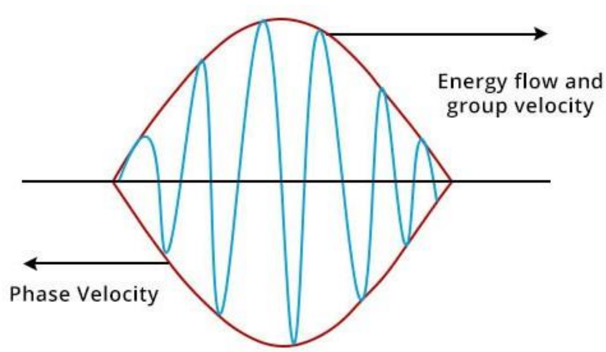
In the case of a fixed tension T and vibrating length l, the frequency changes inversely with the square root of the mass density (μ).

$$f \propto \frac{1}{\sqrt{\mu}}$$

$$\Rightarrow f = \frac{B}{\sqrt{\mu}}, \text{ where B is a constant}$$

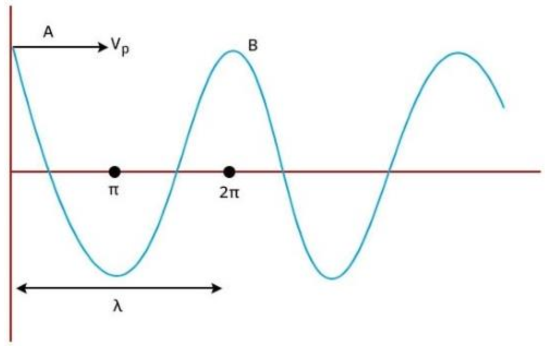
5.8. Phase Velocity and Group Velocity

A wave packet is a collection of waves with a finite spatial extent and a well-defined waveform as a whole. The group velocity is the speed at which a wave packet travels. It shows the speed at which the wave packet's greatest energy or amplitude travels across space. Put differently, it signifies the speed at which the complete set of waves seems to move together as a single unit. The position of a point in the wave cycle at a instant time represent the wave's phase. The speed at which its phase travels defines phase velocity of a wave. It represents the speed at which a particular, constant-phase point—like the crest or trough—moves.



Group velocity and phase velocity have a directly proportional relationship. They are important for comprehending the characteristics of wave propagation. Researchers obtain a thorough understanding of wave events and their applications in other scientific disciplines by examining these connected velocities.

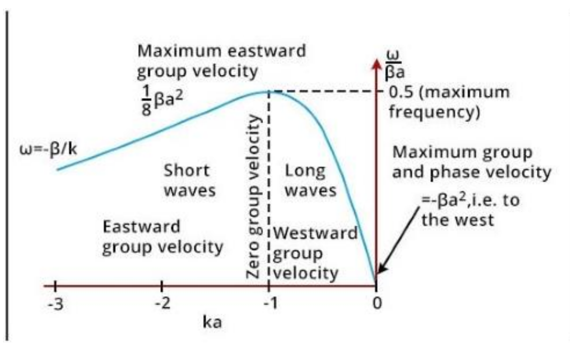
5.9. Phase Velocity



When a wave moves in space or within a wave packet, its phase velocity is the speed at which it does so.

Consider a wave moving through a material or over empty space. A wave's phase, or where it is with relation to a certain reference point, is defined for each point on the wave front. The phase velocity, which carries the phase with it, indicates the speed at which these points travel. It basically signifies the rate at which the wave "appears" to move. When examining a particular point, A, on the wave in the preceding picture, the phase velocity that it moves at is expressed mathematically as "Vp=ω/k."

5.10. Group Velocity



The entire wave packet envelope shape velocity is described by the group velocity. This describes the overall motion of the wave packet over space. Representing the wave's amplitude as it moves across space aids in understanding this idea. According to quantum physics, a "wave packet" is created by fusing together several waves of different frequencies and amplitudes to create a composite waveform with unique properties.

The group velocity can vary greatly from the phase velocity, which is something else entirely. When waves with varied frequencies and wave numbers superimpose within a wave packet, group velocity behaviour results. As such, the group velocity offers important insights on the information and energy transfer inside the wave packet. It establishes the manner in which the wave's energy and properties are carried via the amplitude and envelope of the wave packet.

5.11. Relation between Group Velocity and Phase Velocity

The following formula represents the mathematical link between group velocity and phase velocity:

$$\mathbf{V_g = V_p + k\frac{dV_p}{dk}}$$

where,

V_g, $V_p$are group and phase velocity.

The wave number, or k, is a basic parameter that characterizes a wave's spatial variation. It can also mean the wave's phase change rate per unit of distance.

The group and phase velocity are directly proportional to each other, implying the following:

• A rise in phase velocity corresponds an increment in group velocity.

• Group velocity and phase velocity are directly proportional.

The direct correlation between group velocity and phase velocity is demonstrated by this relationship.

5.12. Equation between Group Velocity and Phase Velocity

Consider the wave packet's amplitude and let ω represent its angular velocity, which is provided by $\omega=2\pi f$, and k represent its angular wave number, which is given by

$$\mathbf{k = \frac{2\pi}{\lambda}}$$

where, t, V_p and V_g is time, phase velocity and group velocity

The phase velocity (V_p) of a wave is given by,

$$\mathbf{V_p = \frac{\omega}{k}}$$

After rewriting this equation, we obtain:

$$\mathbf{\omega = kV_p}$$

Differentiate it w.r.t k we find,

$$\frac{d\omega}{dk} = V_p + k\frac{dV_p}{dk}$$

The group velocity V_g is given by,

$$V_g = \frac{d\omega}{dk}$$

This get reduced to:

$$V_g = V_p + k\frac{dV_p}{dk}$$

The phase and group velocity are correlated, as shown by the equation above. It illustrates how these two velocities relate to one another for a progressive wave.

5.13. Applications of Phase Velocity and Group Velocity

The scientific domains of many different fields use phase and group velocity. Understanding phase and group velocities is crucial for applications in classical physics, including electromagnetic, optics, and acoustics. They help to explaining the phenomena including as interference, diffraction, and refraction. The design and operation of lenses, optical fibers, and other wave-guiding devices depend heavily on the manipulation of phase and group velocities in optics.

Phase velocity and group velocity are very important concepts in quantum physics. They are employed in the study of matter waves, including electrons and other particles with wave-like characteristics, as well as the duality between waves and particles. The real particle velocity and the probability density transport are related to the group velocity of a particle wave packet. Additionally, group velocity may be understood and controlled in telecommunications to ensure low dispersion and signal loss and effective data transfer through optical fibers.

5.14. Vibrating String Energy

Stretched under tension T, a vibrating string has elastic potential energy in addition to kinetic energy of motion. In the nth mode of vibration, the displacement of the string is provided by

$$y_n(x, t) = A_n \sin\frac{n\pi x}{L}\cos(\omega_n t + \emptyset)$$

A string, fixed at both ends, i.e. y = 0 at x = 0 and x = L, the given solution is valid. Additionally, if we define t = 0 as the point at which the string reaches equilibrium means y(x,t) = 0 for all x, then we obtain

$$\cos \varnothing = 0 \quad \text{or} \quad \varnothing = \pi/2$$

Hence, consider the solution

$$y_n(x, t) = A_n \sin \frac{n \pi x}{L} \sin \omega_n t$$

$$\text{Therefore, } \frac{\partial y_n}{\partial t} = A_n \omega_n \sin \frac{n \pi x}{L} \cos \omega_n t$$

$$\text{and, } \frac{\partial y_n}{\partial x} = \frac{n A_n \pi}{L} \cos \frac{n \pi x}{L} \sin \omega n t$$

At location x, the kinetic energy of an element dx of the string is now

$$dK_n = ½ (\mu\, dx) \left(\frac{\partial y_n}{\partial t}\right)^2$$

$$= ½ \mu A_n^2 \omega_n^2 \cos^2 \omega_n t \sin^2 \frac{n \pi x}{L} dx$$

The total kinetic energy at time **t**, in **nth** mode, is therefore

$$K_n = \int dK_n = ½ \mu A_n^2 \omega_n^2 \cos^2 \omega_n t \int_0^L \sin^2 \frac{n \pi x}{L} dx$$

$$= ¼\, M A_n^2 \omega_n^2 \cos^2 \omega n t$$

When the string is shifted from its equilibrium position, the current potential energy of element dx is equal to the work done by the tension T in stretching the specified element dx of the string to length ds :At the spot x, the kinetic energy of an element dx of the string is now

Where, $M = \mu L$ is the string's total mass.

$$dU_n = T (ds - dx)$$

Energy Transfer

• A variety of energy exchanges take place constantly in a range of ordinary situations.

• Common scenarios include the following:

o An object is propelled upward;

o A moving object collides with an obstruction;

o An object is pushed by a continuous force;

o A vehicle accelerates or decelerates;

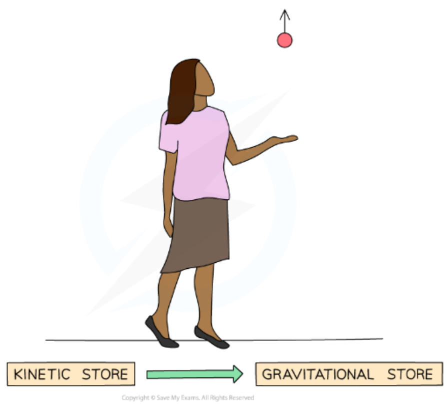
o Water in an electric kettle reaches a boil.

5.15. An Object Projected Upwards

• The ball carrier has energy stored in their chemical store prior to the ball being sent skyward.

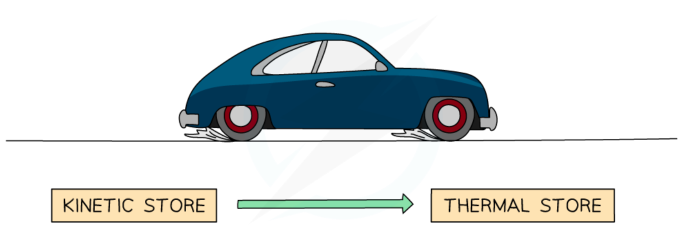
• A portion of the energy is transferred to the ball's kinetic store when it is thrown, causing it to start moving upward.

• Kinetic energy is transformed in to gravitational potential energy as the ball's height increases.



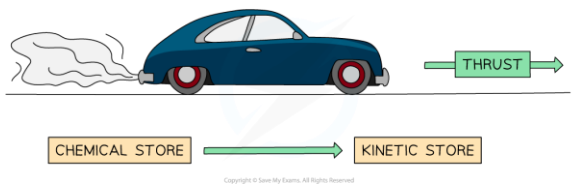
5.16. A Moving Object Hitting an Obstacle

- Energy in the fuel's chemical store is transferred to the vehicle's kinetic store while an object, like an automobile, is moving.
- The speed of the object will rapidly reduce if it encounters an impediment, like a wall. Consequently, the amount of energy in its kinetic store will diminish.
- In this case, the majority of the energy is wasted and transferred to the thermal store of the environment. Energy is mechanically transferred to the thermal store of the wall by the force of the car on the wall.
- In addition, sound waves carry energy out of the system by heating the car's thermal storage, the wall, and the air's thermal store (causing the air particles to vibrate)



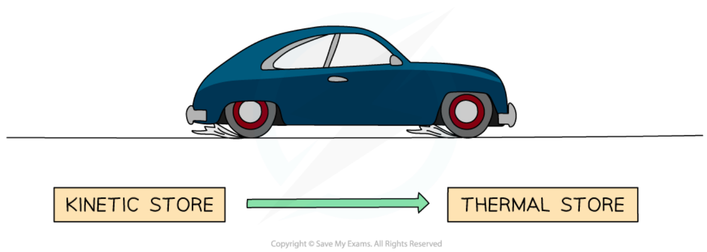
5.17. A Vehicle Accelerated by a Constant Force

- A vehicle's chemical fuel reserve contains energy when it is stationary.
- Energy is transferred to the car's kinetic store when it accelerates or speeds up.



5.18. A Vehicle Slowing Down

- A moving vehicle has energy stored in its kinetic store.
- Energy is transmitted to the surrounding thermal store when it slows down or decelerates (dissipated).
- Heat is generated as a result of friction between the ground-contacting tires and the brake pads, which transfers energy.
- In addition to heating, sound waves also move energy out of the system by causing the air particles to vibrate.



5.19. Boiling Water in a Kettle

- • Energy is transported from the mains to the thermal store of the heating element inside an electric kettle when it boils water.
- • Heat is transmitted from the heating element to the water's thermal store as it gets hotter.

Summary

Understanding the superposition of harmonic waves, the formation of standing waves, and the dynamics of energy transfer within vibrating strings provides insights into the behavior of wave systems and their applications in various fields, including acoustics, music, and engineering. Phase velocity is the velocity at which a point of constant phase moves through space. It can change with position and time due to factors like dispersion. Group velocity is the velocity at which the overall shape or envelope of a wave packet moves through space. It can also vary with position and time.

Keywords

Interference, Diffraction, Superposition, Phase velocity

Objective Type Questions

1. In a standing wave on a string with fixed ends, at which points are the amplitudes of vibration maximum?

   A) Nodes

   B) Antinodes

   C) Points halfway between nodes and antinodes

   D) Points at the ends of the string

2. What happens to the phase velocity of a wave in a string as it travels from a region of higher tension to a region of lower tension?

   A) It increases

   B) It decreases

   C) It remains constant

   D) It becomes zero

3. Which of the following best describes the group velocity of a wave in a string?

   A) It represents the speed at which energy is transferred by the wave.

   B) It represents the speed at which individual particles in the string move.

   C) It is always greater than the phase velocity.

   D) It is the same as the phase velocity in all cases.

4. In a standing wave on a string with free ends, how many antinodes are there?

   A) One

   B) Two

C) Three

D) Four

5. How does the energy of a vibrating string change as the frequency of vibration increases?

A) Zero

B) Constant

C) Negative

D) none of these

Self Assessment

1. What is superposition?

2. What is harmonic waves ?

3. Define group velocity

4. What is phase velocity?

5. Write down the relation between the phase and group velocity.

Chapter 6

Concept of Normal Modes in Stretched Strings

Objectives

1. Understand the concept of normal modes in stretched strings and their significance in wave phenomena.
2. Explore the normal modes of stretched strings generated by plucking and striking methods.
3. Analyze the formation of longitudinal standing waves in stretched strings and their characteristics.

6.1 Normal Modes of Stretched Strings

Consider the situation in which a taut string is fastened at both ends. We now aim a continuous sinusoidal wave at a particular frequency in the direction of positive x. The wave returns and moves to the left end after hitting the right fixed end. In this phase both the waves overlaps which are flowing in opposite direction. This phenomenon occurs again when the wave that is heading leftward reflects off the left end and begins to move right, overlapping the wave that is moving leftward. As a result, numerous overlapping waves are produced, interfering with one another. Standing waves can result from such a situation involving both longitudinal waves, like sound waves, and transverse waves, like waves on the surface of water.

6.2 Equation of Standing Wave:

It is assumed that there are two waves at every given point u and time t, one of which is going to the left and the other left. A representation of the wave moving along the x-axis in a positive direction is as follows:

$$y_1(u,t) = a\sin(ku - \omega t)$$

The wave moving in the negative c direction expressed as:

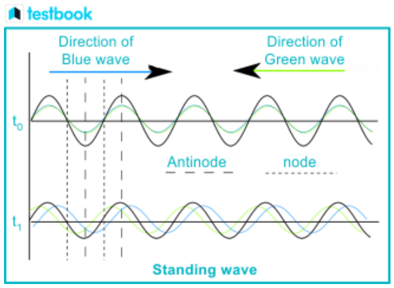
$$y_2(u,t) = a\sin(ku + \omega t)$$

From the superposition principle, the combine wave is represented as:

$$\boldsymbol{y(u,t) = y_1(u,t) + y_2(u,t)}$$

$$y(u,t) = a\sin(ku - 0\omega t) + a\sin(ku + \omega t)$$

$$y(u,t) = 2a\sin(ku)\cos(\omega t)$$



6.3 Nodes and Antinodes:

- A standing wave's node is the location where the wave has the least amount of amplitude.
- On the other hand, an antinode is the location of the standing wave's maximum amplitude. Usually, these are located halfway between the nodes.

6.4 Normal Mode:

A mass preset on a spring oscillate like a wave with single frequency. A stretched string with fixed ends, however, is capable of oscillating at a variety of frequencies and in different patterns. Standing waves or normal modes are the special "Modes of Vibration" of a string.

6.5 Plucked and Struck Strings:

Many musical instruments used worldwide in a variety of musical genres are fundamentally based on string vibrations. The wave equation is the fundamental mathematical formula that describes vibration on strings.

Wave phenomena of all kinds are described by variations on this fundamental equation, ranging from waves on the ocean to electromagnetic waves that carry information from our radios and televisions, ultrasonic waves that are used to create images inside our bodies, and electromagnetic microwaves that boil our food.

The air vibrations in a tube, such as those seen in brass and woodwind instruments, are described by the same equation.

Three main factors influence the frequency at which the strings of musical instruments that are plucked or bowed will vibrate:

1. The string's length (L), which the player can shorten with their fingers on some instruments (guitar, violin).

2. mass density of the string (μ)

3. The string's tension (T), which performers modify to "tune" their instruments

Resonant frequencies are the frequencies at which a string vibrates most frequently when it has a particular combination of L, T, and μ. These resonant frequencies, which are determined by the following equation, are almost identical to the natural frequencies for a string in air:

$$f_n\,(Hz) = \frac{n}{2L}\sqrt{\frac{T}{\mu}} \quad where\ n = 1, 2, 3, \ldots\ and\ so\ on.$$

For a given string, there are truly an endless number of natural frequencies, or fn. Every one of these inherent frequencies corresponds to a specific string motion pattern, sometimes referred to as a mode of vibration or mode shape.

The pattern (modes) and corresponding frequencies that emerge now depend on where and how the string is driven—that is, whether it is plucked or bent.

6.6 Plucked Strings:

In simple terms, plucked strings are pushed sideways at a specific point before being released. Which modes of vibration and related natural frequencies are excited depends on the form of the string when it is released. Any initial deflection pattern of the string can be created by combining the mode shapes shown in the demonstration above. What would happen if the string was pulled precisely in the middle? Explain using below image:

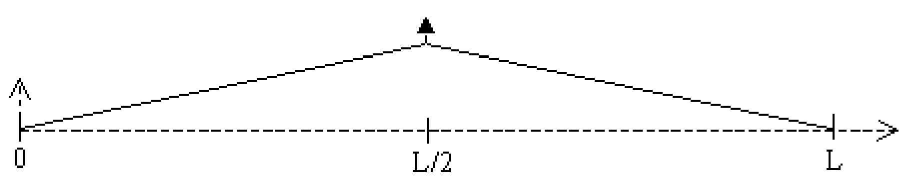


Figure 6.1 : Wave propagation in string

Find the mode of vibration in the above figure 6.1.

It is clear that although the modes work together to produce the initial deflection form that is depicted above, the string does not maintain a tight bend in the middle when it vibrates up and down.

The reason for this is that each of these modes, or shape patterns, vibrates at a different frequency. In addition, you could observe that two pulses seem to be leaving the excitation point (plucked point) in the animation and then reflecting at the ends of the strings. An alternative perspective would be to see it as waves that are oscillating down the string.

The Fourier series can help to establish the set of mode shape combinations required to produce a specific initial deflection shape. A Jean-Baptiste Joseph Fourier (1768–1830) a French mathematician gives Fourier series, which is initially employed to create a theory on the conduction of heat.

The guitar string is actually plucked close to one end, by the bridge. The diagram's approximate place could be at x = L/5.

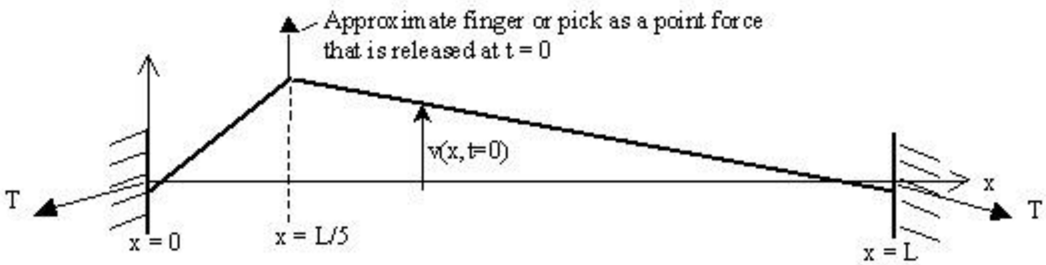


Figure 6.2 vibrations in struck string

6.7 The Struck String

1. A nonzero initial velocity distribution and zero initial displacement are characteristics of an ideal struck string.
2. A striking string with rigid terminations at both ends (and no losses) is simulated for velocity wave motion in the above image.

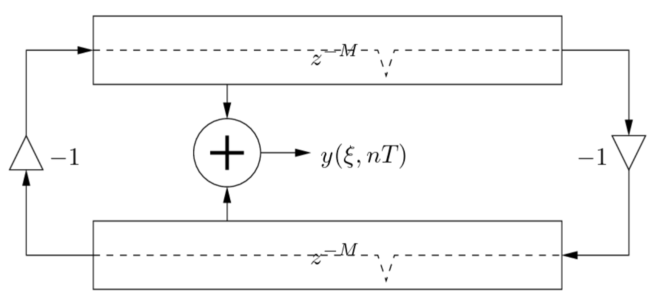


Figure 6.3: Representation of ideal wave propagation (Lossless) on a string fixed at its both ends simulated using a digital waveguide velocity wave (with strike initiation).

- The simulation of velocity waves is more realistic for the impacted string. On the other hand, for determining the equivalent initial displacement, the initial velocity distribution integrated with respect to x from x=0, and divided by c, and negated in the upper rail.
- The graphic below (for the strike initialization shown above) simulates wave motion in a struck string that is rigidly ended at both ends (and without loss).

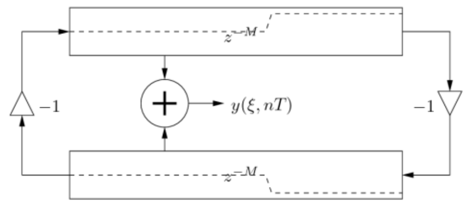


Figure 6.4: Digital waveguide displacement-wave simulation of ideal lossless wave propagation on a string fixed at both ends (with strike initialization).

6.8 Longitudinal Wave

When the displacement of the medium aligns with the direction of the travelling wave, the wave is said to be longitudinal.

The wavelength (λ) indicates the separation between the centers of two successive compression or rarefaction zones. When two waves' compression and rarefaction regions line up, it is called constructive interference; when they don't it defines destructive interference.

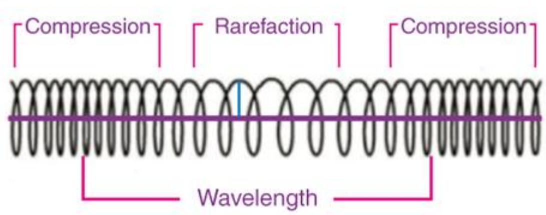


Figure 6.5 Longitudinal Wave

A longitudinal wave's compression zone is where the particles are closest to one another, and its rarefaction zone is where the particles are farthest apart.

6.9 Longitudinal Wave Formula

$$y(x,t) = y_0 \cos[\omega(t - xc)]$$

Where,

- Y ,x ,t, y_o, c and ω are displacement, distance the point travelled, time, amplitude of the oscillations, speed of the wave and angular frequency of the wave respectively.

6.10 Sound Waves

One type of longitudinal wave is sound, which is created when particles move across a conductive medium and start to vibrate. The tuning fork is an illustration of longitudinal sound waves.

The amplitude of a wave in sound waves is the difference between the air's undisturbed pressure and the wave's maximum pressure. The kind of medium, its composition, and the temperature at which sound travels all affect how quickly sound spreads.

Figure 6.6 Sound Wave

6.11 Pressure Waves

The definition of a pressure wave is the disturbance that spreads through a medium as the pressure changes.

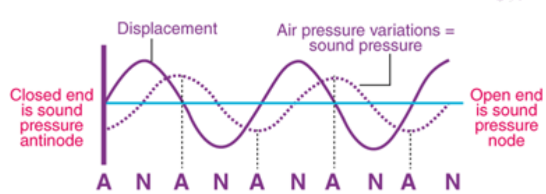


Figure 6.7 Pressure Wave

Representation of wave equation for a harmonic pressure wave,

$$y(x,t) = y_0 \cos(kx - \omega t + \varphi)$$

Here y, x, t, y_o, ω and φ are displacement, distance the point travelled, time, angular frequency and phase difference of the wave respectively.

6.12 Longitudinal waves characteristics:

Compression

It is a zone where the wave's particles are nearly close to with each other.

Rarefaction

In longitudinal wave, rarefaction occurs when the particles are most separated from one another.

Wavelength

Wavelength of a longitudinal wave is the separation between two successive points. The intervals between these locations can be either compressions or rarefactions.

Amplitude

The particle's greatest displacement from its rest position is known as its amplitude. The amplitude of a longitudinal wave is the distance to compression or rarefaction from the medium's equilibrium position.

6.13 Period and Frequency

The term "period" refers as, how long it takes a wave to travel one wavelength and frequency of a wave means wavelengths per second.

6.14 Distinction between Longitudinal and Transverse wave

Longitudinal Wave	**Transverse Wave**
A wave that travels along its path of propagation	A wave that travels perpendicular to its path of propagation
Example: Sound wave.	Example: Water waves.
There are refractions and compressions occurs in longitudinal wave	Troughs and Crests are occurs in transverse wave
This wave can be generated in any kind of media, including solid, liquid, or gas.	The surface of both solids and liquids can generate this wave.

Summary

Understanding the concept of normal modes in stretched strings, their generation by plucking and striking, and the formation of longitudinal standing waves provides insight into the behavior of vibrating systems and their applications in music, acoustics, and engineering. The fundamental frequency of the standing wave corresponds to the lowest resonant frequency of the string, where the string vibrates as a single segment.

Keywords

Nodes, antinodes, Standing waves, Wave number

Objective Type Questions:

1. What are the normal modes of a stretched string?

A) Patterns of vibration that result from constructive interference

B) Specific patterns of vibration with frequencies determined by the length, tension, and linear density of the string

C) Random patterns of vibration that occur when a string is plucked or struck

D) Standing waves that only occur at certain frequencies

2. What happens when a string is plucked at its center?

A) Odd harmonics produced

B) Even harmonics produced

C) Both harmonics are produced

D) No produced

3.The fundamental frequency of a vibrating string is related to as ?

A) Tension in string

B) length of string

C) Material of string

D) Amplitude of vibration

4. When a string is struck, what determines the frequencies of the resulting normal modes?

A) Force of strike

B) Amplitude of vibrations

C) Shape of string

D) Boundary conditions of string

5. What term is used to describe the longitudinal standing waves that form in a column of air in a pipe?

A) Normal modes

B) Harmonics

C) Overtones

D) Nodes and anti

Self Assessment

1. What are normal modes in stretched strings
2. What is Longitudinal waves
3. What is standing waves
4. Define Harmonics
5. Define Tension.

Chapter 7

Simple Harmonic Motion

Objectives

1. Understand the concept of simple harmonic motion (SHM) and its characteristics.
2. Explore the differential equation governing SHM and learn how to solve it.
3. Analyze the kinetic and potential energy associated with objects in SHM and understand the concept of total energy.

Introduction

A restoring force that is exactly proportionate to the body's displacement from its mean position is called, "**Simple Harmonic Motion (SHM)**".This restorative force always towards in the direction of the mean position.

In simple harmonic motion, Accelerationis $\alpha(t) = -\omega^2 x(t)$. where ω is the angular velocity of the particle.

Simple harmonic motion is a special case of oscillatory, not "all oscillatory motions are Simple Harmonic Motions (SHM), although all oscillatory motions are periodic and oscillatory". All oscillatory motions are collectively referred to as harmonic motions, with simple harmonic motion (SHM) being the most significant type.

Considerate the properties of sound waves, light waves, and alternating currents can be greatly aided by studying simple harmonic motion. A superposition of many harmonic motions at various frequencies can be used to describe any oscillatory motion that is not simple harmonic.

7.1 Difference between a periodic motion, oscillation and simple harmonic motion

Periodic Motion:

- After every equal interval of time, a motion repeats itself, called periodic motion.
- No equilibrium position exist.
- In this motion restoring force does not exist.
- Stable equilibrium position also does not exist.

Oscillation motion:

- Particle moves into and fro motion about a mean position.
- This motion is periodic motion when bounded between two extreme points. Ex: Spring-mass pendulum

- The mean position, also known as the equilibrium position, is the location along any path (the path is not a constraint) at which the item will continue to move between two extreme positions about a in variant point.
- The mean position, sometimes referred to as the equilibrium position, will be the target of a restoring force.
- At the mean position in such motion, the particle experiences no net force.
- A stable equilibrium position is the mean position.

Simple Harmonic Motion (SHM)

- Together with a straight line connecting the two extreme locations, it is a particular instance of oscillation (the path of SHM is a constraint).
- A force will be applied in the direction of the mean position that is called restoring.
- Mean position represents a stable equilibrium.

7.2 Necessary condition for SHM

$$\vec{F} \propto -\vec{x}$$

$$\vec{a} \propto -\vec{x}$$

7.3 Two types of SHM

1. Linear SHM
2. Angular SHM

1. Linear SHM-

A particle is said to be moving in linear simple harmonic motion when it travels in a straight line to and from a fixed points. For example, spring-mass system.

The restoring force or acceleration applied to the particle must always be directed toward the equilibrium position and proportionate to the particle's movement.

$$\vec{F} \propto -\vec{x}$$

$$\vec{a} \propto -\vec{x}$$

Here, $\vec{x}$denotes displacement from equilibrium position; $\overrightarrow{F}$ restoring force and $\vec{a}$ acceleration

2. **Angular SHM**

When a system oscillates angularly long with respect to a fixed axis, it is said to be in angular simple harmonic motion.

The angular acceleration or restoring torque applied to the particle must always be proportionate to its angular displacement and aim for the equilibrium position.

$\mathbf{T \propto \theta}$ **or** $\mathbf{\alpha \propto \theta}$

Here, T, α and θ are the torque, angular acceleration and angular displacement respectively.

Key Terms of SHM

3. **Mean position**

$$\vec{F} \propto -\vec{x}$$

$$\vec{a} \propto -\vec{x}$$

4. **Conditions at mean positions**

$$\overrightarrow{F_{net}} = 0$$

$$\vec{a} = 0$$

The force acting on the particle is negative of the displacement. So, this point of equilibrium will be a stable equilibrium.

Amplitude in SHM

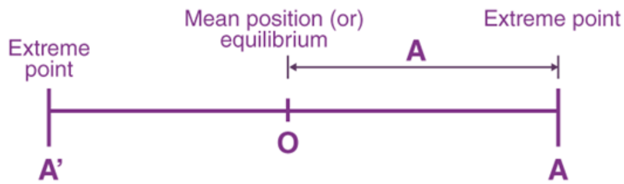


Figure 7.1

In mean position, particle have the maximum displacement

Time period of SHM

The time period is the shortest period of time used to complete one oscillation.

T = 2π/ω

Frequency of SHM

The frequency is the number of oscillations per second, mathematically, f = 1/T and

Angular frequency ω = 2πf = 2π/T

Phase of SHM

The condition of an oscillating particle with respect to its displacement and direction of vibration at any given time is known as its phase. The particle's expression and location in relation to time .x is equal to x = A sin (ωt + Φ).

At time t=0, the phase of the particle is called as the initial phase.

Phase Difference in SHM

The phase difference is the difference between the total phase angles of two particles. When two vibrating particles have an even multiple of π as their phase difference, they are said to be in the same phase.

$\Delta\Phi = n\pi$ where n = 0, 1, 2, 3, ...

If there is an odd multiple of π in the phase difference between two vibrating particles, they are said to be in opposing phases.

$\Delta\Phi = (2n + 1)\,\pi$ where n = 0, 1, 2, 3, ...

7.4 Equation & solution of SHM

Consider a particle of mass m, moving along a path $X^{/}OX$, and its mean position is Oas shown in figure 7.2 (a). When the particle is at p point its velocity is v_0 and then position is X_0 (t=0) (i.e., particle moving in right side) and Q position when t=t and at this point particle having velocity is v.

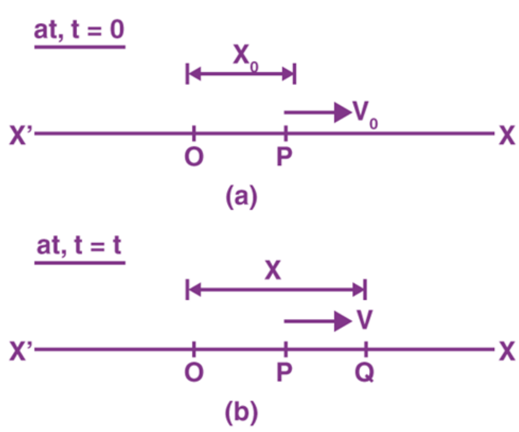


Figure 7.2

The restoring force $\vec{F}$ at Q is given by

$\vec{F} = -K\vec{x}$

K – is a positive constant

$\Rightarrow \vec{F} = m\vec{a}$

Here,

$\vec{a}$ – acceleration at Q

$\Rightarrow m\vec{a} = -K\vec{x}$

$\Rightarrow \vec{a} = -\left(\frac{K}{m}\right)\vec{x}$

$Put\ \frac{K}{m} = \omega^2$

$\Rightarrow \omega = \sqrt{\frac{K}{m}}$

$\Rightarrow \vec{a} = -\left(\frac{K}{m}\right)\vec{m} = -\omega^2\vec{x}$

Since,

$\left[\vec{a} = \frac{d^2x}{dt^2}\right]$

$\frac{d^2\vec{x}}{dt^2} = -\omega^2\vec{x}$

$d^2x/dt^2 + \omega^2 x = 0$, which is the differential equation for linear Simple Harmonic Motion.

Solutions of Differential Equations of SHM

The differential equation for the Simple Harmonic Motion has the following solutions:

- $x = A\sin\omega t$
  (This solution when the particle is in its mean position point (O) in figure (a)
- $x_0 = A\sin\phi$
  (When the particle is at the position & (not at mean position) in figure (b)
- $x = A\sin(\omega t + \phi)$
  (When the particle at Q at in figure (b) (any time t).

These solutions can be verified by substituting these *x* values in the above differential equation for the linear simple harmonic motion.

Conditions for an Angular Oscillation to be Angular SHM

The body must experience a net torque that is restored in nature. If the angle of oscillation is small, this restoring torque will be directly proportional to the angular displacement.

T ∝ − θ

T = − kθ

T = Iα

α = − kθ

$$I\frac{d^2\theta}{dt^2} = -K\theta$$

$$\frac{d^2\theta}{dt^2} = -\left(\frac{K}{I}\right)\theta = -\omega_0^2\theta$$

$$\frac{d^2\theta}{dt^2} = -\omega_0^2\theta = 0$$

This is the **differential equation of an angular Simple Harmonic Motion**. The solution of this equation is the angular position of the particle with respect to time.

$$\theta = \theta_0 \sin\left(\omega_0 t + \phi\right)$$

Then the angular velocity,

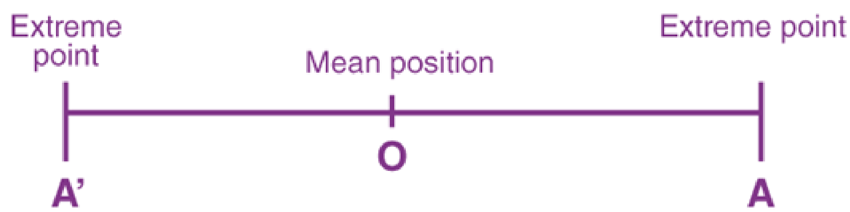
$$\omega = \theta_0 .\ \omega_0 \cos\left(\omega_0 t + \phi\right)$$

θ_0 − Amplitude of the angular SHM

7.5 Example:

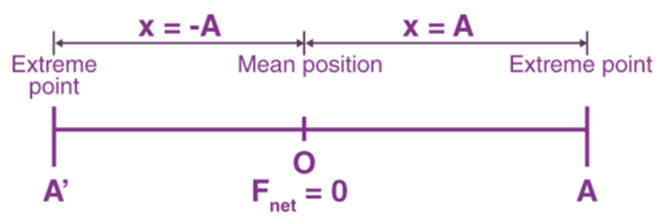
- Simple pendulum
- Compound pendulum
- Physical pendulum
- Torsional oscillator

SHM Quantitative Analysis



Assume for the moment that a particle is moving in Simple Harmonic Motion between points A and A1, passing through the equilibrium position (O) or mean position (A). Here is an examination of it.

SHM in O point



Displacement	x = -A	x = 0	x = +A
Acceleration	|a| = Max	a = 0	|a| = max
Speed	|v| = 0	|v| = Max	|v| = 0
Kinetic energy	KE = 0	KE = Max	KE = 0
Potential energy	PE = Max	PE = Min	PE = Max

Necessary Conditions for Simple Harmonic Motion

- $\vec{F} \propto -\vec{x}$
- $\vec{a} \propto -\vec{x}$
- $\vec{a} = -\omega^2 x$
- $\vec{a} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$
- $\vec{a} = v\frac{dv}{dx} = -\omega^2 x$
- $\int_0^v v dv = \int_0^x -\omega^2 x dx$
- $\frac{v^2}{2} = \frac{-\omega^2 x^2}{2} + c \ldots .(1)$

Some conditions we know:

At point A v = 0 [x = A] the equation (1) becomes

$\frac{v^2}{2} = \frac{-\omega^2 A^2}{2} + c$
Using, v = 0

$0 = \frac{-\omega^2 A^2}{2} + c$
$c = \frac{\omega^2 A^2}{2}$
Sub the value of C in equation (1)

$\frac{v^2}{2} = \frac{-\omega^2 x^2}{2} + \frac{\omega^2 A^2}{2}$
$\Rightarrow v^2 = -\omega^2 x^2 + \omega^2 A^2$
$\Rightarrow v^2 = \omega^2 \left(A^2 - x^2\right)$
$v = \sqrt{\omega^2 \left(A^2 - x^2\right)}$
$v = \omega\sqrt{A^2 - x^2} \ldots . (2)$
Where v is the velocity of the particle executing simple harmonic motion from the definition of instantaneous velocity

$v = \frac{dx}{dt} = \omega\sqrt{A^2 - x^2}$

$\Rightarrow \int \frac{dx}{\sqrt{A^2 - x^2}} = \int_0^t \omega dt$
$\Rightarrow \sin^{-1}\left(\frac{x}{A}\right) = \omega t + \phi$
x = Asin (ωt + Φ) . . . . . (3)

Equation (3) – Equation of the position of a particle as a function of time.

Case 1: If at t = 0

The particle at x = x0

$\Rightarrow \sin^{-1}\left(\frac{x}{A}\right) = \omega t + \phi$
$\Rightarrow \sin^{-1}\left(\frac{x_0}{A}\right) = \phi$
Φ is the initial phase of the particle.

Case 2: If at t = 0

The particle at x = 0

$\sin^{-1}\left(\frac{O}{A}\right) = \phi$
i.e. Φ = 0

Case 3: If the particle is at one of its extreme positions, x = A at t = 0

$\Rightarrow \sin^{-1}\left(\frac{A}{A}\right) = \phi$
$\Rightarrow \sin^{-1}(1) = \phi$
⇒ π/2 = Φ

So, the value can be anything depending upon the position of the particle at t = 0. That is why it is called the initial phase of the particle.

Now, if we see the equation of the position of the particle with respect to time,

π/2 = x = A sin (ωt + Φ)

sin (ωt + Φ) is the periodic function, whose period is T = 2π/ω

Which can be anything, sine function or cosine function

Now, if we see the equation of the position of the particle with respect to time,

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Which can be anything, sine function or cosine function

Time Period of SHM

The coefficient of t is ω.

So, the time period $T = 2\pi/\omega$

$\omega = 2\pi/T = 2\pi f$

ωt = Angular frequency of SHM

From the expression of particle position as a function of time:

We can find particles, displacement $\left(\overrightarrow{x}\right)$, velocity $\left(\overrightarrow{v}\right)$ and acceleration as follows.

The Velocity of a Particle Executing Simple Harmonic Motion

Velocity in SHM is given by $v = dx/dt$,

$x = A \sin(\omega t + \Phi)$

$$v = \frac{d}{dt} A \sin(\omega t + \phi) = \omega A \cos(\omega t + \phi)$$
$$v = A\omega\sqrt{1 - \sin^2\omega t}$$

Since $x = A \sin \omega t$

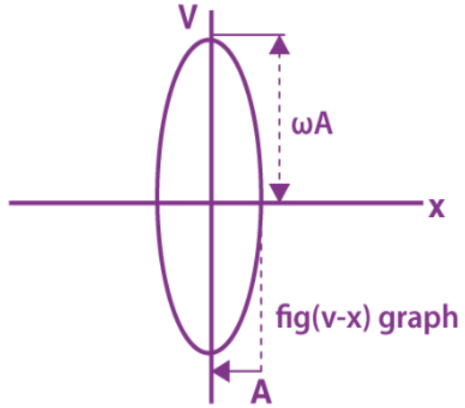
$$\frac{x^2}{A^2} = \sin^2\omega t$$
$$\Rightarrow v = A\omega\sqrt{1 - \frac{x^2}{A^2}}$$
$$\Rightarrow v = \omega\sqrt{A^2 - x^2}$$

On squaring both sides,

$$\Rightarrow v^2 = \omega^2\left(A^2 - x^2\right)$$
$$\Rightarrow \frac{v^2}{\omega^2} = \left(A^2 - x^2\right)$$
$$\Rightarrow \frac{v^2}{\omega^2 A^2} = \left(1 - \frac{x^2}{A^2}\right)$$
$$\Rightarrow \frac{v^2}{A^2} + \frac{v^2}{A^2\omega^2} = 1$$

this is an equation of an ellipse.

The curve between displacement and **velocity of a particle executing the simple harmonic motion is an ellipse.**



When ω = 1, then the curve between v and *x* will be circular.

Acceleration in SHM

$\vec{a} = \frac{dv}{dt} = \frac{d}{dt}(A\omega\cos\omega t + \phi)$
$\Rightarrow \vec{a} = -\omega^2 A\sin(\omega t + \phi)$
$\Rightarrow |a| = -\omega^2 x$

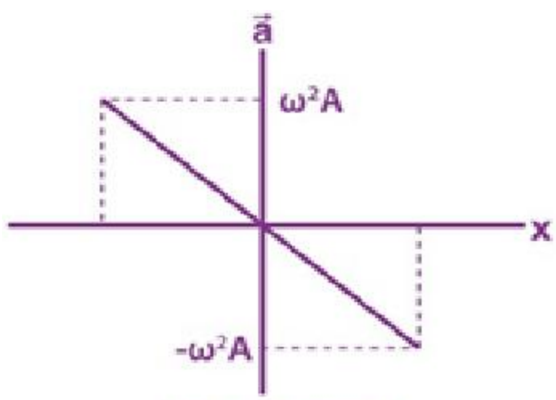


fig. $\vec{a}$ - $\vec{x}$ graph

Energy in Simple Harmonic Motion (SHM)

The system that executes SHM is called the harmonic oscillator.

Consider a particle of mass m, executing linear simple harmonic motion of angular frequency (ω) and amplitude (A),

the displacement $\left(\overrightarrow{x}\right)$, velocity $\left(\overrightarrow{v}\right)$ and acceleration $\left(\overrightarrow{a}\right)$ at any time t are given by

x = A sin (ωt + Φ)

$v = A\omega\cos\left(\omega t+\phi\right) = \omega\sqrt{A^2-x^2}$

$a = -\omega^2 A\sin\left(\omega t+\phi\right) = -\omega^2 x$

The restoring force $\left(\overrightarrow{F}\right)$ acting on the particle is given by

F = -kx, where $k = m\omega^2$.

Kinetic Energy of a Particle in SHM

Kinetic Energy

$= \frac{1}{2}mv^2\ \left[Since,\ v^2 = A^2\omega^2\cos^2\left(\omega t+\phi\right)\right]$

$= \frac{1}{2}m\omega^2 A^2\cos^2\left(\omega t+\phi\right)$

$= \frac{1}{2}m\omega^2\left(A^2-x^2\right)$

Therefore, the Kinetic Energy

$= \frac{1}{2}m\omega^2 A^2\cos^2\left(\omega t+\phi\right) = \frac{1}{2}m\omega^2\left(A^2-x^2\right)$

Potential Energy of SHM

The total work done by the restoring force in displacing the particle from (x = 0) (mean position) to x = x:

When the particle has been displaced from x to x + dx, the work done by restoring force is

dw = F dx = -kx dx

$$w = \int dw = \int_0^x -kx dx = \frac{-kx^2}{2}$$
$$= -\frac{m\omega^2 x^2}{2}$$
$$\left[k = m\omega^2\right]$$
$$= -\frac{m\omega^2}{2} A^2 \sin^2(\omega t + \phi)$$

Potential Energy = -(work done by restoring force)

$$= \frac{m\omega^2 x^2}{2} = \frac{m\omega^2 A^2}{2} \sin^2(\omega t + \phi)$$

Total Mechanical Energy of the Particle Executing SHM

E = KE + PE

$$E = \frac{1}{2} m\omega^2 \left(A^2 - x^2\right) + \frac{1}{2} m\omega^2 x^2$$
$$E = \frac{1}{2} m\omega^2 A^2$$

Hence, the particle's total energy in SHM is constant, independent of the instantaneous displacement.

⇒ Relationship between kinetic energy, potential energy and time in Simple Harmonic Motion at t = 0, when x = ±A.

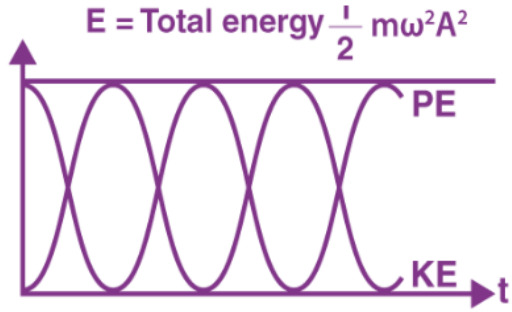
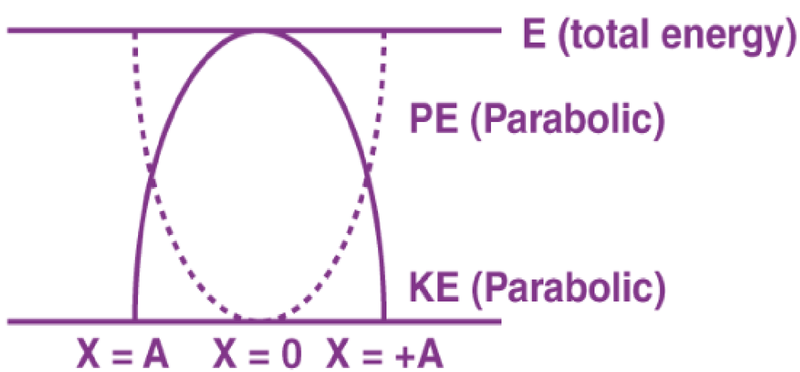


fig. (PE-KE-t) graph

⇒ Variation of kinetic energy and potential energy in Simple Harmonic Motion with displacement.



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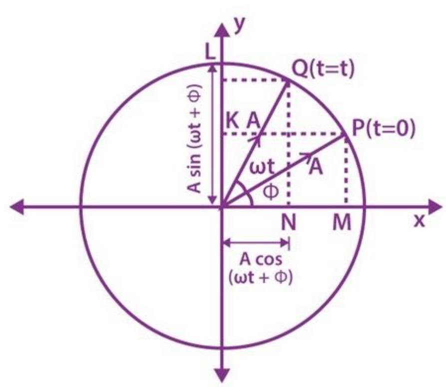
8. Geometrical Interpretation of SHM

A particle moving at a constant speed around a circle has simple harmonic motion, which is represented by the straight line motion of the foot of the perpendicular drawn from the particle on the circle's diameter.

9. SHM as a Projection of Circular Motion



The particle is at position P at t = 0 and revolves along a circle with a constant angular velocity (ω). The projection of P on the diameter along the x-axis (M). At the later time (t), the particle is at Q. Now, its projection on the diameter along the x-axis is N.

As the particle P revolves around in a circle anti-clockwise, its projection M follows it up, moving back and forth along the diameter, such that the displacement of the point of projection at any time (t) is the x-component of the radius vector (A).

$x = A \cos (\omega t + \Phi)$ . . . . (1)

$y = A \sin (\omega t + \Phi)$ . . . . . (2)

Thus, we see that the uniform circular motion is the combination of two mutually perpendicular linear harmonic oscillations.

It implies that P is under uniform circular motion, (M and N) and (K and L) are performing simple harmonic motion about O with the same angular speed ω as that of P.

P is under uniform circular motion, which will have centripetal acceleration along A (radius vector).

$\overrightarrow{a_c} = A\omega^2$
(towards the centre)

It can be resolved into two components:

- $a_N = A\omega^2 \sin^2(\omega t + \phi)$
- $a_L = A\omega^2 \cos^2(\omega t + \phi)$

a_N and a_L acceleration correspond to the points N and L, respectively.

In the above discussion, the foot of the projection on the x-axis is called a horizontal phasor.

Similarly, the foot of the perpendicular on the y-axis is called the vertical phasor. We already know the vertical and horizontal phasor will execute the simple harmonic motion of amplitude A and angular frequency ω. The phases of the two SHMs differ by π/2.

Summary

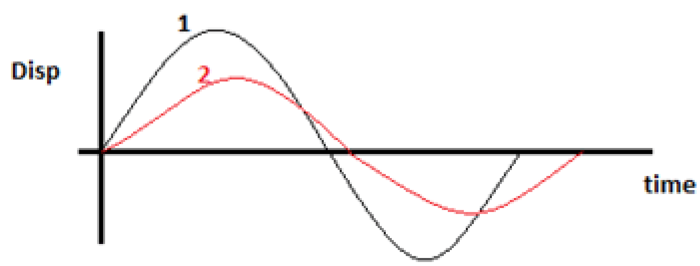
Understanding simple harmonic motion, its differential equation, solution, and the concepts of kinetic, potential, and total energy provides insight into the behavior of oscillatory systems and their applications in various fields, including physics, engineering, and biology.

Keywords

Periodic motion, Angular frequency, phase constant, Oscillation

Objective type question

1. At the highest position in SHM has zero value
    a) Distance
    b) Velocity
    c) Acceleration
    d) None of these
2. The displacement vs time graphs of 2 SHMs are given below. Which parameter is the same for both of them?



    a) Angular frequency
    b) Amplitude
    c) Maximum speed
    d) Phase constant
3. In simple harmonic motion, Phase difference between acceleration and velocity
    a) Zero
    b) 180^0
    c) 90^0
    d) 45^0

4. A particle has an equation of motion given by: $x = \cos^2 wt - \sin^2 wt$.The true statement is the same.

a) Not SHM

b) SHM with $T = \pi/w$

c) SHM with $T = 2\pi/w$

d) Motion amplitude is $1/\sqrt{2}$ m

5. Damping force on a spring mass system is proportional to

a) Wavelength

b) Momentum

c) Velocity

d) Force

Self Assessment

2. What is kinetic energy
3. Define potential energy
4. What is SHM?
5. What is mean position?
6. Write down the equation of motion.

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Chapter 8

Damped and forced harmonic oscillations

Objectives

1. Understand the behavior of damped harmonic oscillators and forced harmonic oscillators.

2. Explore the differential equations governing damped and forced harmonic oscillations and learn how to solve them.

3. Analyze the solutions of the differential equations in different cases, including under damped, critically damped, and over damped scenarios.

4. Investigate the concepts of logarithmic decrement, power dissipation, sharpness of resonance, and quality factor associated with damped harmonic oscillations.

8.1 Introduction

Two all too typical examples of oscillations are simple pendulums and alternating current. The period, frequency, and amplitude of an oscillation are some of the variables that control it. Generally speaking, an oscillation is a movement that alternates between two points in a regular rhythm. In this essay, let's examine oscillation in more detail.

8.2 Simple Harmonic Motion

Since this is the most fundamental kind of oscillation, we all regularly experience this kind of oscillatory motion. It is an oscillatory motion in which the amount of retarding force given to an object determines how far it moves from its equilibrium position. Stated differently, the restoring force is proportionate to the displacement and acts in the opposite direction of the object's displacement.

A pendulum in motion offers another straightforward example, where the restoring force works in the opposite direction of displacement when the pendulum is displaced in one direction. Any simple harmonic motion falls into one of three forms of oscillation:

These are explained individually below.

8.3 Main Three Types of Simple Harmonic Motion

- Damped oscillation
- Driven oscillation
- Free oscillation

A periodic change in time of a substance about its mean value or between two fixed states is referred to as oscillation in physics.

A cycle of periodic to and fro motion of the body about its central position in space is referred to as oscillation. In a closed system, vibrations are mechanical oscillations. A particle oscillates between two locations that are near to its center when it vibrates. The height or greatest distance that an oscillation can occur across is known as its amplitude. It is stated in terms of meters.

The frequency is the number of full oscillation cycles that occur in a second, whereas the time period is the length of time it takes to complete one oscillation cycle. Seconds are used to measure the duration. Frequency can be defined as the reciprocal of time period. The frequency is expressed in hertz, while the time interval is expressed in seconds. In hertz, the frequency is expressed.

8.4 Distinctiveness of Oscillation

• The number of complete oscillations that take place in a single unit of time is known as frequency.

• The largest movement of an oscillator from its equilibrium point is known as its amplitude.

• The time period, expressed in seconds, is the length of time required for one full oscillation to occur.

• The formula $f=1/T$ represents the relationship between frequency and period.

Example

Once more, a straightforward pendulum's motion makes a great example of simple harmonic motion. This is due to the fact that a proportionate restoring force operates on the pendulum in the opposite direction when it is moved in one direction.

Three important groups can be distinguished from simple harmonic motions according to the external factors at work. The following are these categories: Next, we will define oscillating motion and go over its several types. Each type will then be briefly discussed.

8.5 Free Oscillation

When there is no external resistance affecting a particle's motion, as it usually does, it is said to be in a free oscillation state. It is a motion with constant amplitude, energy, and time period, as well as a natural frequency that matches the particle. It is an ideal state because, in the real world, all oscillating objects interact with their surroundings in some way and experience a net loss of energy. This is not the case in this ideal state.

Absence of an external force to initiate the oscillation results in a constant amplitude and period for the free oscillation. In perfect conditions, damping does not occur in free oscillation. All natural systems do, however, exhibit damping, and this will remain the case unless an ongoing external force is given to overcome the damping. The energy, frequency, and amplitude in such a system are all maintained at the same levels.

A basic pendulum in a vacuum is an example of a free oscillation.

8.6 Damped Oscillation

The oscillation is categorized as damped based on the energy difference between the applied restorative force and the restraining force acting on it. In essence, it is an oscillation that gradually disappears, which implies that both the oscillations' frequency and intensity would diminish.

There are two types of damping: artificial and natural.

The simplest illustration of damped oscillations is the oscillation of a basic pendulum in its natural state. Automobile shock absorbers are one type of dampening equipment that is used to lessen vibrations in the vehicle.

The several forms of damped oscillation are as follows:

1. The damping constant is one under damped oscillations.
2. Critically damped oscillations: Damping constant equal to unity.
3. When the damping constant is more than 1, oscillations are said to be over damped.

8.7 Forced Oscillation

The term "forced oscillation" refers to oscillations that occur in a body due to the action of an external periodic force. The external energy that is given to the system causes the oscillation's amplitude to dampen but stays constant.

To maintain the swing from losing its effectiveness, you must push the person on it repeatedly at regular intervals.

Conclusion

The oscillating system is given resistance, which is recognized as damping. A damped oscillation is an oscillation that gets smaller with time.Damping causes oscillations to become less pronounced over time. Amplitude is reduced when the system loses energy in overcoming external forces like air resistance or friction in addition to other resistive factors. Consequently, the system's energy falls in tandem with the system's amplitude decline.

8.8 Sharpness of Resonance

Introduction

Understanding resonance makes it easier to express how sharp resonance is. In physics, resonance is very important, when the system's tendency and amplitude both increase with increasing excitation frequency. When the amplitude is lower, the resonance is sharper, and as damping increases, the resonance's sharpness is maximized. There are numerous varieties of resonance in the physical world, such as electrical, mechanical, and acoustic resonance. The definition of resonance, which shows that resonance tends to be acute at a certain point in time, illustrates how sharp resonance may be. One excellent example of mechanical resonance is swing.

8.9 Define Resonance & Sharpness of Resonance

The large selected response of a material or system that vibrates in step or phase with an externally applied oscillation force is known as resonance. It is illustrate as the system’s tendency to move to and fro at the highest amplitude at minimal frequencies in comparison to the others. Since the circuit can either absorb or discharge the maximum amount of energy at the resonance, resonance is known to be particularly important. Radio receivers are one of the practical applications of resonance.

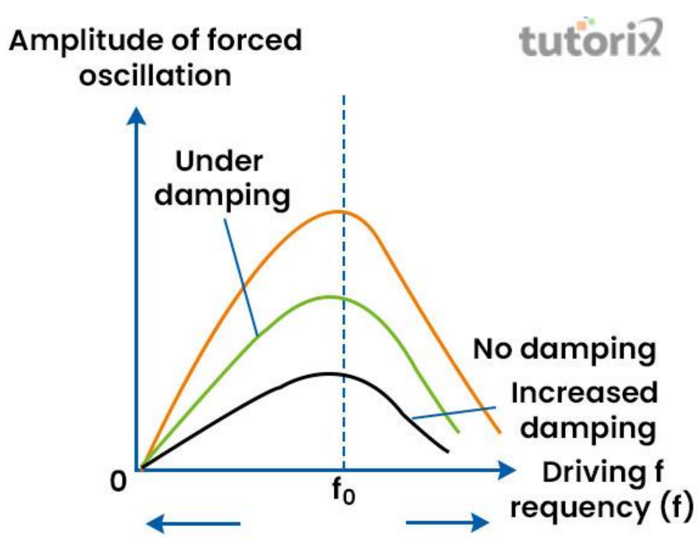


Figure 8.1 Sharpness of resonance

"Sharpness of resonance" mostly depends on two factors which are amplitude and damping. The "sharpness of resonance" is concerned with the factor of Q of the "RLC" circuit. It featured how rapidly the energy perished in the circuit of RLC that is attached to the alternating voltage.

($\omega_0/2\Delta\omega$) quantity is reflected as the calculation of resonance's sharpness. The fact that resonance narrows or sharpens with decreasing $\Delta\omega$ serves as evidence for this. In "Sharpness of resonance", the bandwidth is represented by 2Hω, while the resonance frequency is represented by ω_0. The sharpness of resonance's amplitude is represented by the continuously varying wave height. A consequence in which the wave's amplitude reduces over time is known as damping.

8.10 Illustration of Q Factor

The Q factor, which is the quality factor with no dimensions, is related to resonance sharpness. The under damped resonator, its bandwidth, and center frequency are all featured by the q factor. The Q component in "Sharpness of resonance" can be expressed mathematically as $Q = E_{stored}/E$ lost per cycle. It is determined that the bandwidth of the modulated circuit minimizes when the Q-factor (quality factor) maximizes.

As losses decrease, energy is effectively stored in the circuit, sharpening the modulated circuit. The Q factor establishes a relationship between the maximum energy stored in the circuit's reactance and the energy lost or resisting throughout each oscillation cycle. The term "resonant frequency" ratio refers to the relationship between bandwidth and circuit height, where a narrower bandwidth corresponds to a higher circuit.

8.11 Resonance in LCR Series Circuit

The phenomenon in a circuit occurs when the output of an electric circuit is high at a particular frequency is referred to as resonance. The values of the resistance, conductance, and inductance in the LCR circuit determine the frequency. When the capacitive and inductive reactance have comparable magnitudes but cancel each other out, the series LCR circuit reaches resonance. The distance from the stage is 180 degrees.

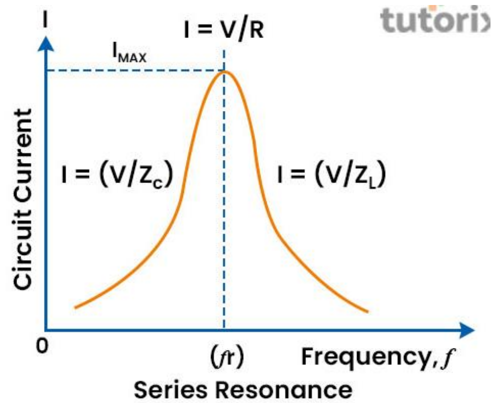


Figure 8.2- Resonance in LCR circuit

The LCR circuit's impedance is illustrated by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Based on the equation, R is known to be resistance, X_L is referred to as inductive resistance and X_C is determined as capacitive resistance. Now, resonance in the LCR circuit is

$$X_L = X_C$$

$$\omega_L = \frac{1}{\omega_C}$$

So, $\omega = \frac{1}{\sqrt{LC}}$ which is demonstrated as the frequency of the resonance.

Moreover, impedance is minimal at the resonance, Z_{min} = R which reflects that current reaches the highest, I_{max} = V_{ms}/R. In the LCR circuit, resonance may take place if the reactances are opposite and similar.

8.12 Power Factor at Resonance

At resonance, the AC circuit and the LCR circuit have different power factors. The LCR circuit functions as a pure resistive circuit at resonance. The impact of the inductor and capacitor cancel each other out. The LCR circuit is thought to be the most functional circuit. It is found that the LCR circuit has a power factor of 1.

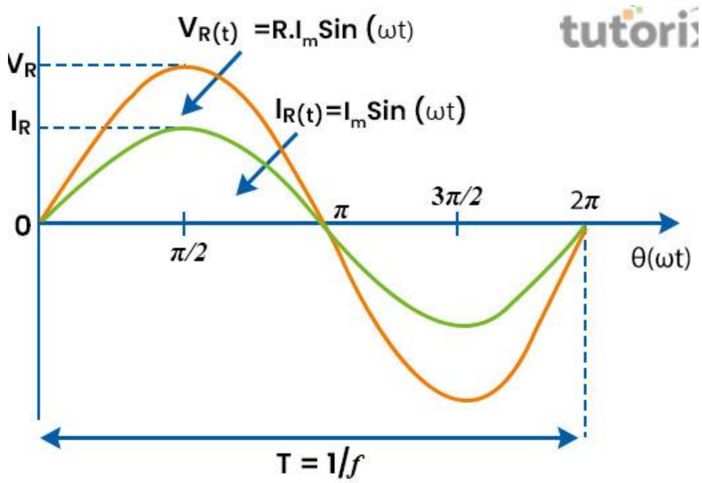


Figure 8.3- Power factor at resonance

At resonance, the inductive reactance resembles the capacitive reactance, indicating that the voltage across the inductor and capacitor has cancelled. The circuit current in the above figure is in a similar phase and in the same direction. As a result, at unity power factor, the phase angle between current and voltage is zero.

Conclusion

The primary characteristic of resonance is the displacement of electrons across comparable atomic nuclei. One example of resonance is when glass breaks and produces a resonant sound. Another example of resonance in an electrical circuit is when various types of resonance occur. It also happens when all the atoms remain in the same plane. Overall, the analysis shows that amplitude is inversely proportional in the case where damping is directly related to resonance sharpness.

8.13 Quality Factor/ Q factor- formulas & Equations

Quality factor, or 'Q', is a term used extensively in RF design and many other fields of electrical circuit design to indicate how well an inductor or tuned circuit performs in a resonator circuit. The quality factor, or Q, is a dimensionless quantity that expresses how well the circuit damps down noise. Additionally, it shows the band width of the resonator in relation to its center frequency.

Quality factor values are frequently stated and can be used to describe how well an inductor, capacitor, or tuned circuit performs.

Many RF-tuned circuits and elements employ the Q, or quality factor, to show how well they operate in an oscillator or other resonant circuit.

It is easy to apply the many straightforward formulas that link the bandwidth and losses to the Q to ascertain different elements of a pertinent circuit.

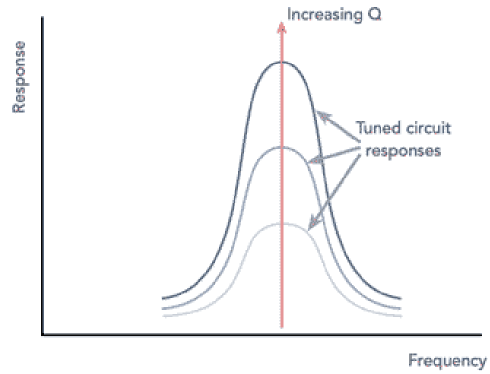


Figure 8.4- Q- quality factor concept for tuned circuits

How can the quality factor develop?

An engineer by the name of K. S. Johnson from the Western Electric Company's engineering department in the US initially conceptualized the idea of Q, or the quality factor. Johnson was assessing the quality and performance of various coils. Throughout his research, he came up with the idea of Q. It's interesting to note that he chose the letter Q for the quality factor—though, in retrospect, there could have been no better choice—because all the other letters in the alphabet had already been taken.

8.14 Basics of Q factor

The idea of quality factor has applications in many branches of engineering and physics. It can also be called the Q factor and is represented by the letter Q. The Q factor is a dimensionless parameter that represents the energy losses in a resonant element, which can be an electronic circuit such as a resonant circuit, a mechanical pendulum, or an element in a mechanical construction. Although an element's Q factor is immediately related to its losses, it also directly correlates to a resonator's bandwidth in relation to its center frequency.

In relation to the total quantity of energy stored in the system, the Q represents energy loss. Because of this, oscillations will decrease more slowly with higher Q values, meaning they will have less damping and ring for longer periods of time. Resistance in electronic circuits is the source of internal energy losses. The inductor is the primary source of resistance, though it can happen anywhere in the circuit.

8.15 Definition of Q factor

It is frequently necessary to define the term "quality factor" in order to provide a more precise comprehension of this quantity. Having a clear definition in short terms is often beneficial.

OR

In electronic circuits, Q is the ratio of the energy that is provided to the resonator by a per cycle to maintain a constant signal amplitude at a frequency where the energy stored in the resonator remains constant throughout time.

Q is a measure of an inductor's efficiency that may alternatively be described as the ratio of its inductive reactance to its resistance at a specific frequency.

In a similar vein, Q is frequently employed in conjunction with capacitors and holds particular significance for vector diodes, as their losses can significantly affect the operation of any voltage-controlled resonant circuit that makes use of them.

8.16 Effects of Q factor-

Q factor matters for numerous reasons when working with RF tuned circuits. While a high level of Q is generally advantageous, certain applications could call for a specific degree of Q.

The following summarizes some of the factors related to Q in RF tuned circuits:

Bandwidth: The tuned circuit filter's bandwidth decreases as the Q factor, or quality factor, rises. The tuned circuit gets sharper as losses go down because energy is better stored in the circuit. It is evident that when the Q rises, the 3 dB bandwidth falls and the tuned circuit's total responsiveness rises. A high Q factor is frequently required to guarantee that the necessary level of selectivity is attained.

Wide Bandwidth: Wide bandwidth operation is necessary for many radio frequency applications. Wide bandwidths are needed for some modulation techniques, but fixed filters are needed for other applications that need wide band coverage. Although there is a need for excellent rejection of undesirable signals, wide bandwidths are also necessary. As a result, in many applications, it is necessary to ascertain the necessary level of Q in order to deliver the overall performance that satisfies specifications for a broad bandwidth and sufficient rejection of undesirable signals.

Phase Noise from Oscillators: Phase noise is produced by all oscillators. This includes erratic changes in the signal's phase. Noise that emanates from the primary carrier is one way that this shows up. It should come as no surprise that this noise is undesirable and has to be reduced.

There are several approaches to modify the oscillator design to decrease this, the main one being to raise the Q, or quality factor, of the oscillator-tuned circuit.

General Spurious Signals: To eliminate spurious signals, tuned circuits and filters are frequently employed. The circuit will be better able to eliminate the spurious signals the sharper the filter and the higher the level of Q.

Ringing - A resonant circuit's losses reduce as its Q rises. It will therefore take longer for any oscillation to dissipate if it occurs within the circuit. To put it another way, the circuit will probably "ring" more. Since less energy is lost in the tuned circuit, this makes it easier to start up and maintain an oscillation, making it perfect for usage in oscillator circuits.

Q factor formulas

The basic Q or quality factor formula is based upon the energy losses within the inductor, circuit or other form of component.

From the definition of quality factor given above, the Q factor can be mathematically expressed in the Q factor formula below:

$$Q = \frac{E_{\text{Stored}}}{E_{\text{Lost per cycle}}}$$

Where:

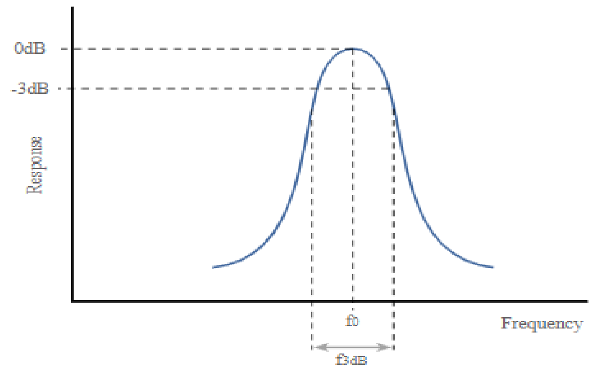
Q = quality factor of the circuit
E_{stored} = energy that is stored
$E_{\text{Lost per cycle}}$ = energy that is lost over each cycle

When looking at the bandwidth of an RF resonant circuit this translates to the Q factor formula:

$$Q = \frac{F_0}{F_{-3\text{dB}}}$$

Where:

Q = quality factor of the circuit
F_0 = centre frequency of the tuned circuit
$F_{-3\text{dB}}$ is the frequency offset from the centre frequency where the response falls by 3 dB.



Q quality factor showing the centre and -3dB points

Within any RF or other circuit, each individual component can contribute to the Q or quality factor of the circuit network as a whole. The Q of the components such as inductors and capacitors are often quoted as having a certain Q factor or quality factor.

8.17 Quality factor and damping

Damping is one feature of the Q factor that matters in a lot of circuits. The qualitative behavior of simple damped oscillators is determined by the Quality Factor, or Q, which also influences the response of other circuits, including filters.

Regarding the damping and Q factor, there are three primary regimes that can be taken into consideration.

Under-damped (Q>1/2)

When the Q factor is more than half, the system is considered under-damped. When a step impulse is delivered, systems with a Q factor of only slightly over half may oscillate once or twice before the oscillation stops. The damping reduces and oscillations will be sustained for longer as the quality factor rises. Without the requirement for additional stimulus, the oscillation would be sustained indefinitely in a theoretical system with an infinite Q factor. While some signal is recycled back into oscillators to offer an extra stimulation, a higher Q factor typically results in a much cleaner output. The signal exhibits lower amounts of phase noise.

Over-damped (Q<1/2)

A system that is over damped has a Q factor that is smaller than half. There are significant losses and no overrun in this kind of system. Rather, after a step impulse is applied, the system will exponentially decrease and asymptotically reach the steady state value. The system reacts to a step impulse more slowly as the Q factor, or quality factor, decreases.

Critical-damped (Q=1/2)

Like an over-damped system, the critically damped system has a Q factor of 0.5 and an output that is neither oscillatory nor overshoots its steady-state output. Without overshooting, the system will reach the steady-state asymptote as quickly as possible.

It is necessary to have high Q factor in many RF resonant systems. Adequate selectivity is necessary for filters, but not excessively so, and high Q values for oscillators lead to better stability and reduced phase noise. In many systems, having a Q factor that is too high could lead to filter bandwidths that are too narrow and oscillators that are unable to track throughout the necessary range. Q factor levels, however, should ideally be high rather than low.

A crucial component of resonant and filter circuits, and particularly in RF design, is the Q, or quality factor. Q can also be significant in other domains, such as general electronic circuit design and certain audio design elements, such as loudspeakers and similar devices.

Gaining a foundational knowledge of Q will facilitate a deeper comprehension of various aspects of circuit design and functioning across multiple domains.

Summary

Understanding damped and forced harmonic oscillations, their differential equations, solutions, and associated parameters such as logarithmic decrement, power dissipation, sharpness of resonance, and quality factor provides insight into the behavior of oscillatory systems in real-world applications, including mechanical engineering, electrical engineering, and physics. Quality factor (Q)) is a dimensionless parameter that measures the sharpness of resonance and is related to the ratio of the resonant frequency to the width of the resonance peak.

Keywords

Resonance, Damping coefficient, Q-factor, Phase shift

Objective Type Questions

1. In SHM, The energy of particle is
   a) Direct proportion to displacement
   b) Direct proportion to square of displacement
   c) Independent of displacement
   d) none of these
2. Two simple motions are represented by
   $y_1=5(\sin 2\pi t+\sqrt{3}\cos 2\pi t)$
   $y_2=5\sin(2\pi t+\pi/4)$
   The ratio of the amplitude of two simple harmonic motions is?

a) 1:1
b) 1:2
c) 2:1
d) 1:$\sqrt{3}$

3. Spring of mass 30kg and spring constant is 15N/m, time period is
   a) 2πs
   b) $2\sqrt{2}\ \pi$s
   c) $2\sqrt{2}\pi$ s
   d) $2\sqrt{2}$ s
4. In SHM, Phase constant represent?
   a) Initial displacement
   b) Initial velocity
   c) Initial acceleration
   d) None of the above

5. For an object undergoing SHM, the maximum kinetic energy occurs when:

a) Displacement is maximum

b) Velocity is maximum

c) Acceleration is maximum

d) Displacement is zero

Self Assessment

1. What is simple harmonic motion?
2. Write down the unit of force ?
3. What Damped Harmonic Oscillations?
4. Define power dissipation
5. What is Quality factor?

Chapter 9
Transport of Energy Along Strings

Objectives

1. Understand the mechanisms involved in the transport of energy along strings.
2. Explore the reflection of waves from free and fixed boundaries and study the associated phenomena such as phase change.
3. Analyze the formation of standing waves and resonance in strings and their implications.

9.1 Reflection of Waves

Definition of Reflection of Waves

When a wave strikes any interface that is a boundary between two mediums, it tends to bounce back with the change in direction. This phenomenon of waves is called the reflection of waves.

Reflection is a phenomenon of the medium. There could be numerous factors that affect reflection and other phenomena as such. Reflection causes certain disturbances in the original energy of the wave. Reflection can easily deteriorate the entire point when it comes to energy change.

There are two types of interfaces:

1. Open boundary – when a particular wave strikes an open boundary interface, it tends to get reflected and refracted. Complete balancing back of the wave is not observed. The open boundary reflection provides easy propagation of the wave; the movement on the boundary is very certain.
2. Closed boundary – it is said to be a secure boundary when there is a rigid boundary that helps to reflect back the entire wave. One of the best examples of closed boundaries is waves striking the wall. The propagation of the wave in a secure boundary is not as swift as when seen in an open limit.

9.2 Reflection on The Closed Boundary or Fixed End Reflection

Reflection on a closed boundary is when you pass a wave on a string that is tied on the wall from one end and free from the other. When a certain incident force hits the wall, it creates pressure on the wall.

By following Newton's third law of motion, the wall or other hand also returns this pressure force in an equal and opposite way. This causes the reflection of the given wave.

You must know that the wall is a rigid boundary, and it won't move; hence there is no generation of the wave at the edge. Therefore, to make it clear, the wave's amplitude at the junction becomes zero.

One might also keep in mind that in such a case, $\varphi=\pi$. The reflected wave and the incident wave at the boundary collapsed with each other and became zero. This is the reason why we say so.

The point to be remembered here is that when wave reflection takes place, which involves a close boundary, there will always be a reversal of the direction of the wave, and the face of the wave could be said to be reversed. Hence the value becomes 180.

9.3 Reflection at an Open Boundary or Free End Reflection

Reflection at an open boundary could be imagined as a wave on the string that is completely free of any attachment. This implies that the wave will travel without any hindrance. In such cases, special pulses at the boundaries are generated. Hence the amplitude of this reflection gets increased. Such amplitude gets at the max level in the open border.

Both the reflected wave and also the incident wave are in the same phase in an open boundary reflection. Hence the value of the difference between the phases becomes $\varphi=0$.

9.4 Law of Reflection

When we talk of the law of reflection regarding the waves when incident light hits a certain medium, there are two possibilities: refraction or reflection. The wave that moves towards the medium is the incident wave. When the incident wave strikes the medium, it can rapidly be absorbed by the medium. When it passes through the medium, the wave changes direction, and there are certain laws on how the wave changes the focus.

Specular reflection

Any reflection from a mirror or similar surface, where all the light waves striking the surface gets reflected back is known as specular reflection. This is the most common type of reflection.

Diffuse reflection

The diffuse reflection is the more practical reflection witnessed. In most cases, the light wave hits the surface medium or mirror at an imperfect angel. The light wave hits the imperfections on the reflecting surface, and gets reflected in certain inferior angles.

9.5 Reflection of light waves and the laws of reflection

When discussing the laws of reflection, there are three major laws related to it. These are:

1. When a light wave strikes a mirror or a reflecting surface, the incident ray forms a certain angle with the normal. The angle between the incident ray and the normal is exactly the same as the angle formed between the normal and the reflected beam. On any surface, the rise would be the same.
2. When light strikes a surface, the incident ray, the reflected ray, and the normal all lie in the same plane.
3. The incident ray and the refracted ray are basically on different sides of the normal.

Cause of Reflection

Reflection is a phenomenon that only happens when the reflecting media is appropriate. There could be various reasons why light is scattered.

1. When the plane reflecting surface is smooth and shiny, reflection can easily occur. On the other hand, depending on the quality of the medium, the energy of light is absorbed or reflected back.
2. When the reflecting surface is not smooth, or is rough, there could be a scattering of light. The laws of reflection are still maintained at every angle.

9.6 Characteristics of the Wave Motion

The main characteristics of the wave motion are:

1. Direction – The direction of the wave after and before reflection is something to be taken care of. The main angle formed between the reflected ray and the incident ray would be 90 degrees, and an angular change is witnessed during the process.
2. The angle of reflection – The rise of reflection becomes very important. When we talk of the main characteristics of a wave, the angle of reflection is the main angle formed between the normal and reflected waves. If this angle is compromised, there is no chance of getting the entire reflected wave back.
3. Frequency – When we talk about the frequency of the wave, it is the different numbers that pass a fixed place under a certain amount. The frequency of the reflected wave is exactly the same as that of the incident wave, and this is a big characteristic to be noted. It determines the number of waves.
4. Speed – The speed of the wave is the main velocity of the wave. It is the speed with which the wave travels.

Conclusion: Above this article, we have mentioned some of the most important points you should know about the reflection of waves. There could be numerous changes in a reflection process depending upon the surrounding where waves travel. A light could be a wave, or it could be a particle. It has dual nature; hence the propagation is highly dependent on the matter.

9.7 Study of Phase Change

The definitions of "change of state" and "change of phase" are same. A shift in heat invariably results in a phase transition. The temperature doesn't fluctuate, though. A solid's temperature rises as a result of its molecules' increased kinetic energy, which is produced by applying heat. Because molecules must partially overcome their cohesive forces in order to move around, melting a solid requires energy. Similar to this, energy is needed to evaporate a liquid since doing so separates the molecules and overcomes molecular attraction forces. However, until a phase transition is finished, there is no change in temperature. That is to say, during phase change, all of the energy given is used to separate the molecules; none of it is used to raise their kinetic energy. Therefore, since the kinetic energy of the molecules doesn't change, its temperature won't rise.

The amount of heat generated or absorbed when a material undergoes a phase transition at a steady temperature (e.g. Its latent heat is the transition from solid to liquid at the melting point or from liquid to gas at the boiling point. Specific latent heat is the amount of heat absorbed or released when a substance's unit mass changes its physical phase at a particular temperature. The melting or boiling point is the steady state temperature at which melting or boiling occurs as shown in figure 9.1.

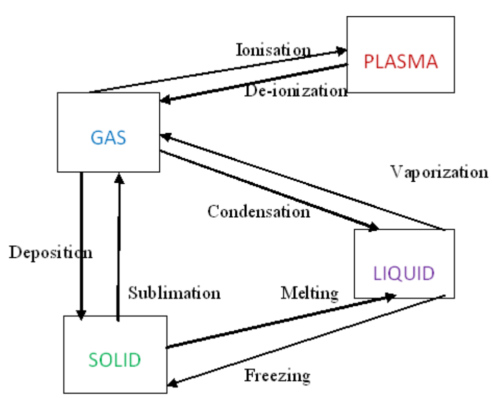


Figure 9.1

Newton's law of cooling governs the phase transition process, stating that "the rate of change of temperature of an object is proportional to the difference between its own temperature and the temperature of its surroundings."

$$\frac{dT}{dt} = -k\ (T - T_a)$$

Here, k is a positive constant; T and T_0 are the object and surroundings temperature respectively.
The melting point, latent heat of fusion, and other properties of a substance can be ascertained by observing its phase transition from solid to liquid.
Examine an example cooling curve for a substance having a melting point of 45^0 C to learn more about the theory of phase change.
At a constant melting temperature of 45^0 C, the liquid-to-solid phase transition is represented by the flat area of the graph. The cooling of the liquid plus tube (left) and the cooling of the solid plus tube (right) are represented by the two curved sections. Newton's law of cooling states that they cool, as shown in figure 9.2.
For a given temperature difference (T-T_0), the heat loss rate of the liquid plus the boiling tube is expected to be the same as the heat loss rate of the solid plus the tube. However, it is unlikely that the material undergoing phase change will have the same specific heat C_2 in the liquid and solid phases. Consequently, we have

$$\frac{dQ}{dt} = (m_1c_1 + m_2c_{2l})\frac{dT}{dt} = -(m_1c_1 + m_2c_{2l})k_l(T - T_0)$$

And

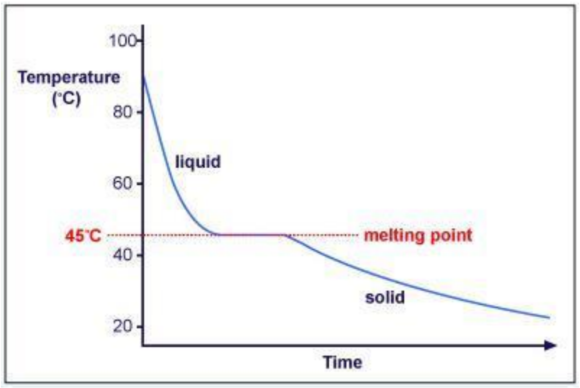
$$\frac{dQ}{dt} = (m_1c_1 + m_2c_{2s})\frac{dT}{dt} = -(m_1c_1 + m_2c_{2s})k_s(T - T_0)$$

Where it is evident that the equation relates the cooling constants in the liquid (l) and solid (s) phases.

$$(m_1c_1 + m_2c_{2l})k_l = (m_1c_1 + m_2c_{2s})k_s$$

By calculating how long it will take the material and tube to cool to 1/e of their initial temperature above room temperature, these cooling constants can be calculated using the graph. Given that the Newton's law of cooling differential equation has a solution that is

$T = T_0 + (T - T_0)e^{-kt}$ we have $k = 1/\ t_e$.



Summary

Energy is transported along strings through the propagation of mechanical waves, which involve the transfer of kinetic and potential energy between adjacent particles in the string. Reflection from a fixed boundary occurs when the wave is not inverted upon reflection, resulting in no phase change. Resonance occurs when the frequency of an external force matches the natural frequency of a system, leading to a significant increase in amplitude and energy transfer.

Understanding the transport of energy along strings, wave reflection, phase change, standing waves, and resonance provides insight into the behavior of waves in mechanical systems and their applications in various fields, including music, acoustics, and engineering

Keywords

Absorption, Transmission, Wave equation, Energy transfer

Objective Type Question

1. How are ultrasonic waves typically detected?

   A) By human hearing

   B) By using specialized microphones

   C) By using infrared sensors

   D) By using radar technology

2. What is the typical wavelength range of ultrasonic waves?

   A) Millimeters to centimeters

   B) Meters to kilometers

C) Micrometers to millimeters

D) Nanometers to micrometers

3. How is the velocity of ultrasonic waves in liquids commonly measured using Sears' method?

A) By measuring the time taken for waves to travel through a known distance in the liquid

B) By analyzing the diffraction pattern produced by the waves

C) By measuring the amplitude of the waves as they travel through the liquid

D) By measuring the frequency of the waves before and after they pass through the liquid

4. In Sears' method, what does the velocity of ultrasonic waves in a liquid depend on?

A) Temperature

B) Density

C) Viscosity

D) All of the above

5. Which of the following is not an application of ultrasonic waves?

A) Medical imaging

B) Non-destructive testing of materials

C) Underwater communication

D) Cooking food in microwave ovens

Self assessment

1. What is refraction?
2. What is phase change?
3. Define resonance
4. Define standing waves
5. Write down the wave equation

Chapter 10

Ultrasonic: Properties and Production

Objectives

1. Understand the properties of ultrasonic waves and their significance in various applications.
2. Explore the methods used for the production of ultrasonic waves, including piezoelectric and magnetostriction methods.

10.1 What is Ultrasonic Wave?

A sound wave is a vibration that travels through a material, like metal, water, or air. "Inaudible sound with high frequency for human" refers to an ultrasonic wave, which typically has a frequency greater than 20 kHz. These days, an ultrasonic wave is another name for a sound wave that is not meant to be heard.

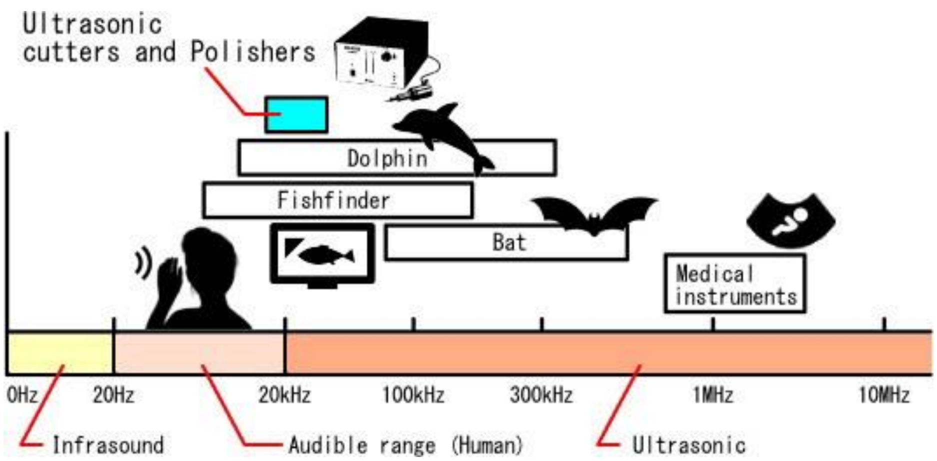


Figure 10.1- Transverse & longitudinal waves with frequencies

Ultrasonic waves come in several forms, such as longitudinal, transverse, and surface waves. Two kinds of elastic waves coexist at the same time in a solid. Two types of elastic waves are known as longitudinal waves or density waves, which have a displacement in the same direction as the wave's propagation direction, and traverse waves or shear waves, which have a displacement in the vertical direction.

The longitudinal wave is what our ultrasonic processing machine mostly uses.

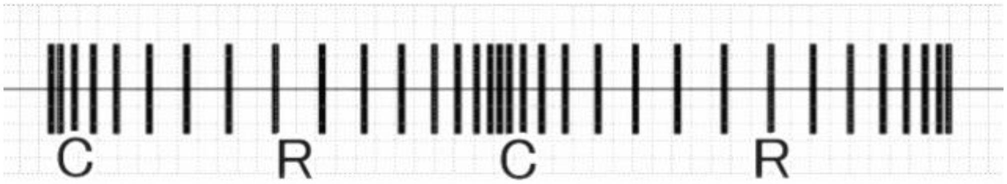


Figure 10.2- Longitudinal wave: Here, C & R represents the rarefaction and compression

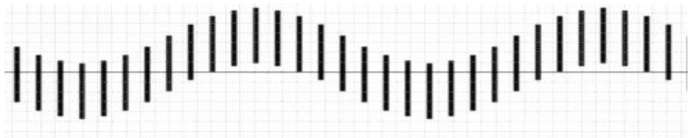


Figure 10.2- Transverse wave-

Table 10.1-Physical properties of longitudinal & transverse wave for some specific materials

Materials	Speed of sound [m/s]	
	Longitudinal wave	Transverse wave
Air	340	-
Water	1500	-
Lead	1960	690
Gold	3240	1220
Iron	5920	3240
Titanium	6100	3120
Aluminum	6380	3130
Beryllium	12890	8880

Characteristics of Ultrasonic Waves:

The frequency range in which sound waves in the human ear are most responsive is 20–20,000 Hz. The name of this range is Audio able range. Ultrasonic sound waves have a frequency of more than 20,000 Hz. There is an audible limit to these frequencies.

These waves move at the speed of sound, which is 330 m/s.

They have a short wavelength.

10.2 Production of Ultrasonic Waves-

Principle

when ferromagnetic material, such as nickel, is magnetized into a rod. It experiences a very slight variation in length longitudinally. We refer to this as the magnetostriction effect.

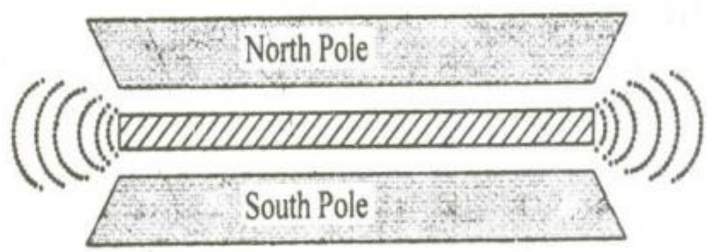


Figure 10.3- Magnetostriction effect.

Construction

Figure 10.4 display the magnetostriction ultrasonic generator's circuit diagram. Between two knife edges is a short nickel rod that is permanently magnetized clamped in the center. On the rod's right hand section, a coil L1 is wrapped. C is a capacitor that varies. The collector-tuned oscillator's resonant circuit is made up of L1 and C1. The base circuit is connected to the coil L2 coiled on the rod's LHS. A feed-back loop is employed with the coil L2.

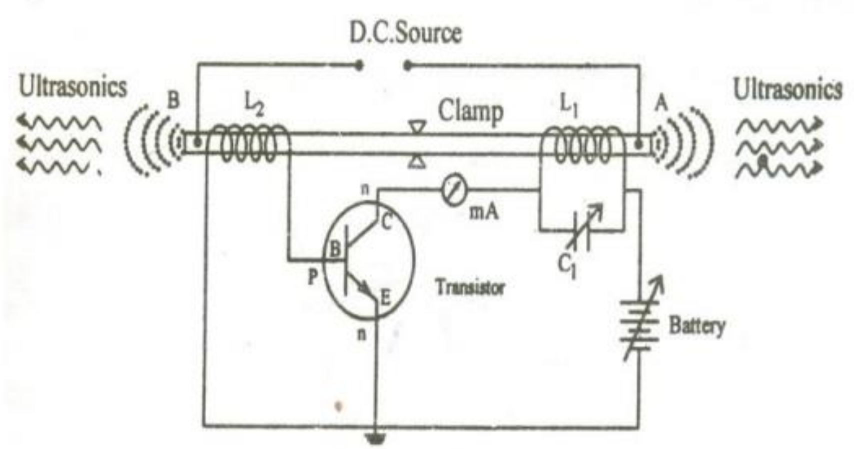


Figure 10.4- Circuit diagram of magnetostriction ultrasonic generator

Working:

The resonant circuit L1C1 creates an alternating current at a certain frequency when the battery is turned on.

$$F = \frac{I}{2\pi\sqrt{L_1C_1}}$$

Throughout the length of the nickel rod, an alternating magnetic field with a frequency of f is produced by the current flowing around coil L1. The magnetostrictive effect causes the rod to begin vibrating. Ultrasonic waves are produced as the rod vibrates.

In the coil L2, the rod's longitudinal expansion and contraction result in an E.M. The transistor's base receives this e.m.f. As a result, positive feedback causes coil L1's high frequency, high oscillation amplitude to grow.

By altering the capacitor, the produced alternating current frequency can be adjusted to match the rod's natural frequency.

Resonance condition- Oscillator circuit frequency equals vibrating rod frequency

$$F = \frac{I}{2\pi\sqrt{L_1C_1}} = \frac{1}{2l}\sqrt{\frac{E}{\rho}}$$

Here, l, E and ρ are represents the length, Young's modulus and density of the rod.

The increase in collector current as reported by the milliampere indicates the resonance state.

Advantages:

1. The magnetostriction Oscillators have a robust mechanical design.

2. The cost of construction is very low.

3. They have a respectable efficiency level and can generate a lot of acoustic power.

Limitations

1. It is limited to producing frequencies of up to 3MHz.
2. The temperature affects the oscillation frequency.
3. The resonance curve has a wide width. It results from the ferromagnetic material's elastic constants vibrating according to its degree of magnetization. Therefore, a steady single frequency is not achievable.

10.3 Piezo Electric Crystals: Principle, Construction, working, Advantages and Disadvantages

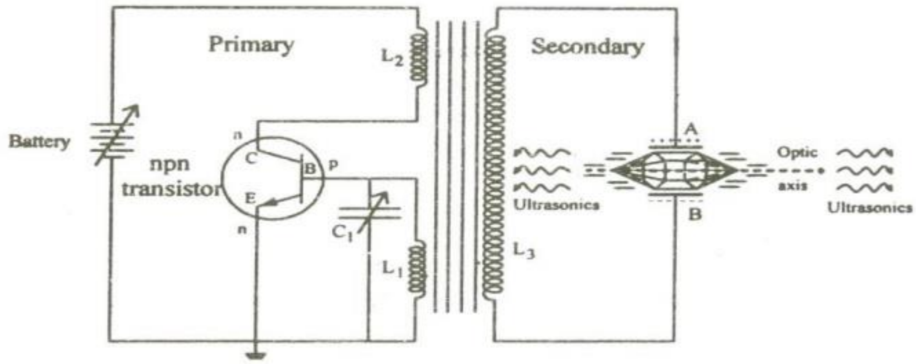


Figure 10.5- Piezo-electric oscillators.

Piezoelectric crystals are those that exhibit both the piezoelectric effect and the converse of it.
Example: Quartz, Tourmaline, Rochelle Salts etc.

A typical example of a quartz piezoelectric crystal is depicted in Figure 10.6. Its shape is hexagonal, with pyramids fastened at either end. It has three axes in total.

i. The optical Z axis connects the pyramid's edges.
ii. The X axis, or electrical axis, connects the hexagon's corners and
iii. The mechanical axis connects the hexagon's sides and center as depicted.

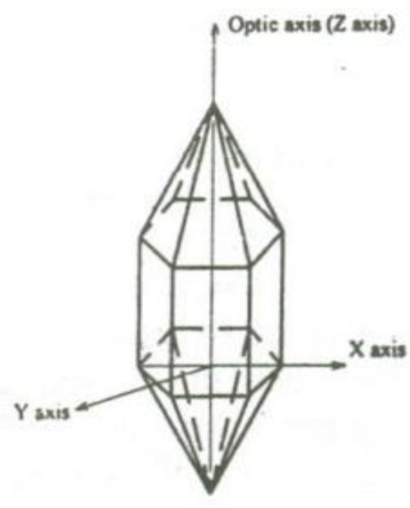
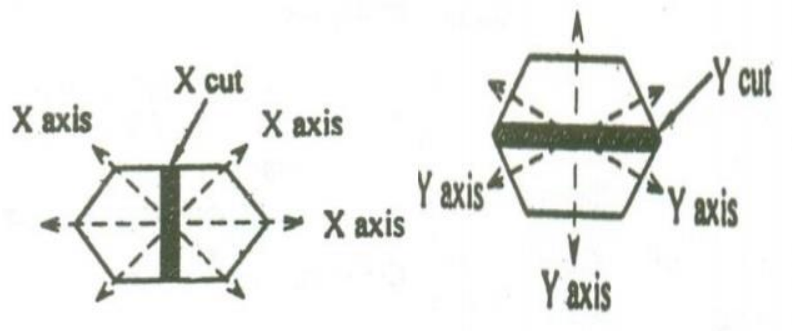


Figure 10.6- Different axis of piezoelectric crystal



(a) **(b)**

Figure 10.7- (a) X-cut and **(b)** Y-cut crystals.

X-Cut Crystal:

The crystal is referred to as an X-crystal when it is cut perpendicular to the X-axis, as seen in figure 10.7 (a). Longitudinal ultrasonic waves are often generated using X-cut crystals.

Y-Cut Crystal:

A crystal is referred to as a Y-cut crystal when it is sliced perpendicular to the Y-axis, as in figure 10.7 (b). Y-Cut crystals typically generate transverse ultrasonic vibrations.

10.4 Piezoelectric Effect

Definition- Expansion or contraction exits in the mechanical axis with respect to the optical axis when alternating electric field is applied to the electrical axis.

10.5 Production of Ultrasonic Waves

A Piezo Electric Effect

Principle:

The inverse piezoelectric effect is the based for this. A quartz crystal experiences elastic vibrations along its mechanical axis when it is exposed to an alternating potential difference along its electric axis. Large amplitude vibrations will occur if the frequency of the electric oscillations matches the crystals inherent frequency. The crystal emits ultrasonic waves if the electric field's frequency falls within the range of ultrasonic frequencies.

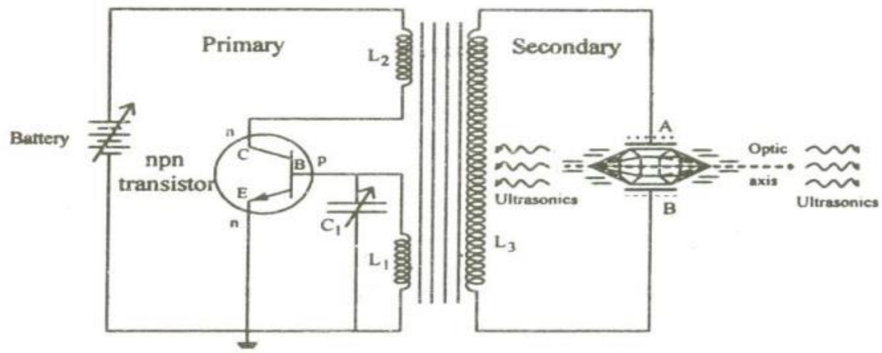


Figure 10.8- Piezo- Electric Oscillators

Construction

Figure 10.8- Displays the circuit diagram. The circuit is a base-turned oscillator. A parallel plate capacitor can be create using a quartz crystal which acts as dielectric, a slice of the crystal is sandwiched between metal plates A and B. This is connected to the electronic oscillator via the transformer's primary coil, L3. The oscillator circuit's coils L2 and L1 are used as the transformer's primary. Base coil L1 and collector coil L2 are inductively linked. The oscillator's tank circuit is composed of the variable capacitor C and coil L1.

Working

High frequency oscillations are produced by the oscillator when the battery is turned on. Transformer operation induces an oscillating e.m.f. in coil L3. So, a high frequency alternating voltage is currently applied to the crystal. To get oscillations that are in resonance with the crystal's inherent frequency, adjust the capacitance of C1. Due to resonance, the crystal is now vibrating with greater amplitude. High strength ultrasonic waves are therefore generated.

Condition for Resonance:

Frequency of the oscillator circuit is equal to frequency of the vibrating crystal.

$$F = \frac{1}{2\pi\sqrt{L_1C_1}} = \frac{P}{2l}\sqrt{\frac{E}{\rho}}$$

Here, l, E and ρ are represents the length, Young’s modulus and density of the rod.

‘P’ = 1,2,3 …. etc for fundamental, first overtone, second overtone etc respectively

Advantages:

1. It is possible to produce ultrasonic frequencies as high as 500MHz.

2. There is a lot of power output. The humidity and temperature have no effect on it.
3. Compared to the magnetostriction oscillator, it is more efficient.
4. The resonance curve has a relatively narrow width. in order for us to obtain ultrasonic waves at a steady and consistent frequency.

Disadvantage

1. Quartz crystal cost is very high.
2. Cutting and molding the crystal is a very intricate process.

Summary

Ultrasonic waves are produced using methods such as the piezoelectric method and the magnetostriction method. In the piezoelectric method, ultrasonic vibrations are generated by piezoelectric materials subjected to an alternating electric field. The magnetostriction method relies on the magnetostrictive effect, where materials change shape when exposed to a magnetic field, to produce ultrasonic waves. Applications of ultrasonic waves are diverse and include medical imaging (ultrasound), non-destructive testing, industrial cleaning, and underwater communication. They are vital in technologies aimed at imaging, inspection, cleaning, and communication in fields such as medicine, industry, and marine exploration.

Keywords

Ultrasonic waves, Attenuation, Transduction, Refraction

Objective Ttype Questions:

1. Ultrasonic waves are defined as sound waves with frequencies:

A) < 20 Hz

B) 20 Hz to 20 kHz

C) >20 kHz

D) <20 kHz

2. What is the primary property that distinguishes ultrasonic waves from audible sound waves?

A) Amplitude

B) Wavelength

C) Frequency

D) Velocity

3. Which method is commonly used to produce ultrasonic waves by converting electrical energy into mechanical vibrations?

A) Piezoelectric effect

B) Magnetostriction effect

C) Electrostriction effect

D) Photoacoustic effect

4. The piezoelectric effect involves the generation of ultrasonic waves through:

A) Application of a magnetic field to certain materials

B) The deformation of certain materials in response to an electric field

C) The deformation of certain materials in response to a magnetic field

D) The application of an electric field to certain materials

5. Magnetostriction is a method used to produce ultrasonic waves by:

A) Applying an electric field to certain materials

B) Applying a mechanical stress to certain materials

C) Applying a magnetic field to certain materials

D) Heating certain materials to high temperatures

Self Assesment

1. What is waves
2. What is sound waves
3. What is supersonic waves
4. What is The piezoelectric effect?
5. What is magnetostriction methods?

Chapter 11

Ultrasonic Waves: Detection and Application

Objectives

1. Understand the principles and methods used to detect ultrasonic waves.
2. Explore the equipment and techniques employed for ultrasonic detection, such as ultrasonic transducers and receivers.
3. Investigate the factors influencing the sensitivity and accuracy of ultrasonic detection systems.

11.1 Ultrasonic waves: Detection and Application

There are several ways to find ultrasonic waves, some of which are mentioned below:

1. **Using radiometer:**

A radiometer can be used to find ultrasonic waves. This approach involves directing an ultrasonic beam onto a thin mica fan that is suspended from one end of a light rod by a thin wire that also holds a small mirror. Ultrasonic waves apply pressure, which causes both the fan and the mirror to be sensed. The placement of the bulb and scale indicates the deflection. A light beam is impinge on the mirror, and the reflected beam returns to the scale's origin on the lamp. The reflected beam on the scale flaws deflects by angle 20 when the mirror exhibits deflection by angle 0. Given that the scale indicates 20, the mirror's deflection can be determined. The ultrasonic wave intensity directly relates to the deflection. Thus, we may use this method to calculate the strength of ultrasonic waves.

2. **Kundt's tube method:**

Lycopodium power is used in Kundt's tube; ultrasonic waves with a wavelength of a few millimeters can also be detected. Secondary waves are created because of the superposition of incident and reflected waves, stationary waves are created when ultrasonic waves travel through a tube. At the node's location, heaps develop. Half the ultrasonic wave wavelength is used to compute the distance between neighboring nodes. Thus, the wavelength of ultrasonic waves can be estimated using this method.

3. **Thermal method for detection of ultrasonic waves:**

A medium produces rare factors and alternate compressions when ultrasonic waves flow across it. The process of compression brings medium particles closer together and increases the likelihood of collisions. This causes the medium's temperature to rise when compressions occur.

Conversely, the medium's temperature drops during rarefaction as a result of the medium's particles moving apart and the frequency of collisions decreasing. Therefore, in the path of ultrasonic wave a platinum resistance thermometer is placed, and move it in the direction of wave propagating, the temperature of thermometer alternately changes. That shows that ultrasonic waves are present in the medium. Sometimes ultrasonic waves that are incident and reflected superimpose to generate stationary waves in the medium. Nodes and antinodes form in the medium in such a scenario. Nodes experience alternate variations in pressure, having a heating and cooling impact. As a result, platinum resistance thermometer added at nodes, store heat, but no temperature change is noted at anodes. As a result, as we move the thermometer within the medium, its platinum wire's resistance changes, demonstrating the presence of ultrasonic waves there.

11.2 Determination of Ultrasonic Velocities in Liquid

Principle

Longitudinal stationery waves are created when ultrasonic waves pass through a transparent liquid as a result of alternating compression and rare action. The liquid exhibits diffraction grating behavior when monochromatic light is transmitted through it perpendicular to these waves. Acoustic grating is the term for this type of grating. In this instance, the rare action and compression lines function as translucent light waves. It is employed to determine the liquid's ultrasonic waves' wavelength and velocity (v).

Construction

It is composed of a liquid-filled glass tank. As seen in figure 1.7, a piezo-electric crystal (Quartz) is installed at the base of the glass tank and coupled to a piezo-electric oscillatory circuit. The monochromatic source (S) in this case is an incandescent bulb, and observed the diffraction pattern using a telescopic configuration. The light within the glass tank is efficiently focused using a collimator made up of the two lenses L1 and L2.

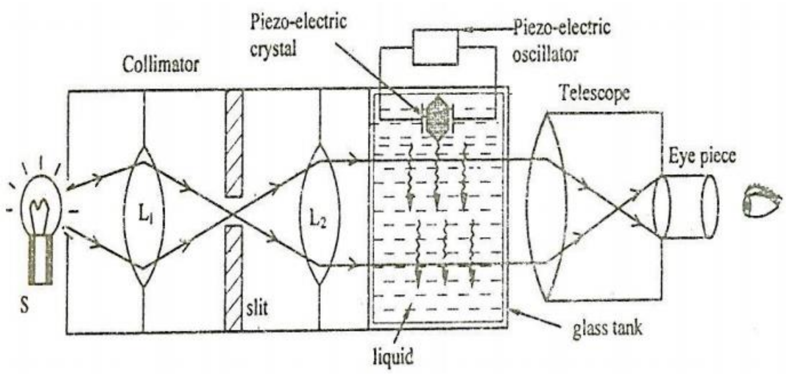


Figure 11.1 - Piezo-electric oscillatory circuit

Working

1. **If the piezoelectric crystal remains stationary**:

At first, the monochromatic light is turned on while the piezoelectric crystal is left at rest. Through the lens, a single picture of a vertical peak is seen when light is concentrated in the glass tank containing the liquid. that is, no diffraction occurs.

2. **As soon as the piezoelectric crystal begins to vibrate:**

Now a piezoelectric oscillatory circuit is used to vibrate the crystal. Ultrasonic waves are generated during resonance and travel through the liquid. The glass tank's walls reflect these ultrasonic waves, which then create a stationary wave pattern in the liquid with nodes and antinodes. Antinodes are where the liquid's density is lowest and nodes are where it increases. As a result, the liquid functions as an auditory grating element, a directing element.

Now that the monochromatic light has been passed, it becomes directed, and as seen in figures 11.2 and 11.3, through the telescope, one may view a diffraction pattern consisting of central maxima and principle maxima on either side.

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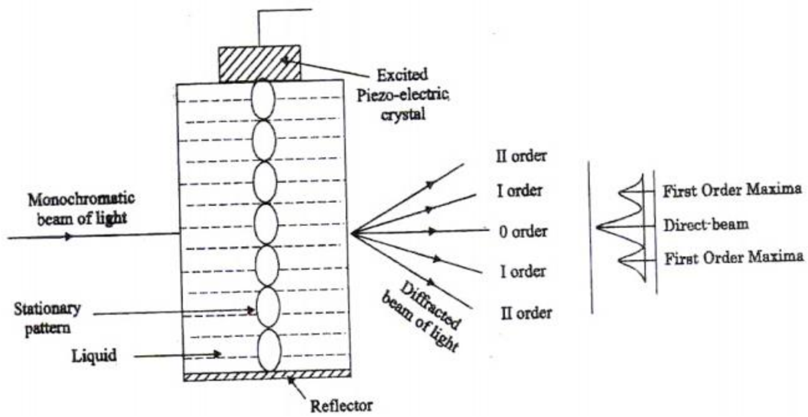


Figure 11.2

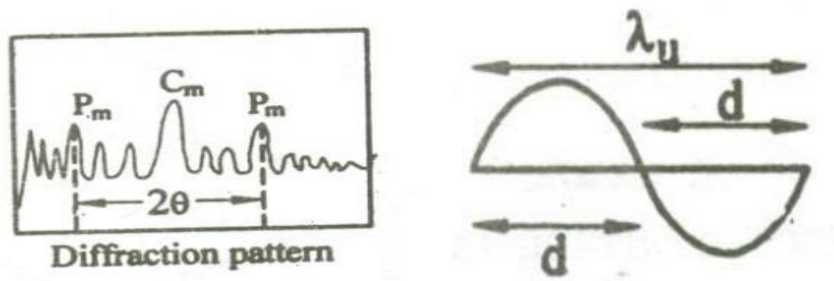


Figure 11.3

Ultrasonic Velocity

By the following conditions we can determine velocity of Ultrasonic waves:

$$2d\sin\theta = n\lambda \longrightarrow \quad (1)$$

Where, d is the distance between successive node or antinodes.

θ is the angle of diffraction

n is the order of the spectrum

λ_L is the wavelength of the monochromatic source of the light.

If $\lambda_u = 2d$ ⟶ (2)

Then, equation (1) becomes,

$$\lambda_u \sin\theta = n\lambda$$

Wavelength of Ultrasonic = $\lambda_u = \dfrac{n\lambda}{\sin\theta}$ ⟶ (3)

We know, Ultrasonic Velocity = Frequency of Ultrasonic × Wavelength of ultrasonic

Velocity of Ultrasonic $v = \lambda_u \times \nu_u$ ⟶ (4)

Substituting equation (3) in (4), we get

Velocity of Ultrasonic $v = \dfrac{\nu_u n\lambda}{\sin\theta}$

11.3 Applications of Ultrasonic Waves

Ultrasonic waves, with frequencies higher than the upper limit of human hearing (typically above 20 kHz), find numerous applications across various fields due to their unique properties. Here are some common applications:

1. **Medical Imaging**: Ultrasonic waves are extensively used in medical imaging techniques like ultrasound. They can penetrate soft tissues and produce detailed images of internal organs, making them invaluable for prenatal care, diagnostics, and monitoring various medical conditions.
2. **Non-Destructive Testing (NDT)**: Ultrasonic testing is used to detect flaws or measure material thickness in manufacturing processes without causing damage to the tested object. This is crucial in industries like aerospace, automotive, and construction for ensuring the integrity of materials and structures.
3. **Cleaning**: Ultrasonic waves are employed in ultrasonic cleaners to remove contaminants from surfaces. The high-frequency waves create cavitation bubbles in a cleaning solution,

which implode near the surface being cleaned, effectively dislodging dirt, grease, and other particles.

4. **Surgery**: Ultrasonic waves are utilized in surgical procedures for cutting, dissecting, and coagulating tissues. Ultrasonic scalpels provide precision and minimize bleeding compared to traditional surgical tools, making them useful in various surgical specialties.
5. **Measurement and Sensing**: Ultrasonic waves are used in various measurement and sensing applications. For example, they are employed in ultrasonic flow meters to measure the flow rate of liquids or gases, and in proximity sensors for detecting the presence or distance of objects.
6. **Sonar**: Ultrasonic waves are utilized in sonar systems for underwater navigation, communication, and detection of objects like submarines, underwater structures, or marine life. Sonar systems emit ultrasonic waves and analyze the echoes to determine the location and characteristics of underwater objects.
7. **Level Sensing**: Ultrasonic sensors are commonly used for level sensing in applications such as tank monitoring, liquid level detection, and object detection. They emit ultrasonic pulses and measure the time taken for the echo to return, which correlates with the distance to the target.
8. **Material Processing**: Ultrasonic waves are employed in various material processing applications, including welding, cutting, and soldering. Ultrasonic welding, for example, uses high-frequency vibrations to create frictional heat, joining materials together without the need for additional adhesives or soldering materials.

Keywords

Ultrasonic transducers, Doppler effect, Sonar, Material characterization

Objective type question

1. Which of the following materials is commonly used in the production of ultrasonic transducers?

a) Copper b) Aluminum c) Piezoelectric ceramics d) Steel

2. What is the principle behind the production of ultrasonic waves in piezoelectric transducers?
a) Magnetic induction b) Electrostatic induction c) Piezoelectric effect d) Photoelectric effect

3. Which of the following applications uses ultrasonic waves for non-destructive testing of materials? a) Ultrasonic cleaning b) Ultrasonic welding c) Ultrasonic imaging d) Ultrasonic flaw detection

4. In medical ultrasound imaging, what property of tissues affects the speed of ultrasonic waves? a) Density b) Elasticity c) Temperature d) Viscosity

5. Which of the following is a common method for detecting the level of liquids using ultrasonic waves? a) Time-of-flight measurement b) Doppler effect c) Reflection measurement d) Absorption measurement

Self assessment

1. Write down the properties of ultrasonic waves
2. Explain the uses of ultrasonic waves
3. Describe the Sear's method
4. Write down the wavelength of ultrasonic waves.
5. How do detect the ultrasonic waves

Chapter 12
Vibrations of Bars

Objectives

1. Studying vibrations of bars, particularly longitudinal vibrations.
2. Understand the behavior of waves propagating through a solid medium like a bar.
3. Exploring the wave equation governing these vibrations and finding its general solution, engineers and physicists can better comprehend how energy travels through materials, aiding in the design and analysis of structures such as beams, rods, and shafts.

12.1 A Wave

A wave is propagating dynamic disturbance form equilibrium; a wave equation can explain its motion. At least two field quantities involved in Physical waves in wave medium. Variables oscillate at a particular frequency around an equilibrium (resting) value on a periodic basis to produce periodic waves. When two superimposed periodic waves move in opposite directions, the resulting standing wave is created; a traveling wave is when the waveform moves in a single direction. There are times when the standing wave's vibration amplitude appears to be diminished or even zero, a phenomenon known as nulls.

12.2 Wave- Definition

A wave is an energy-transporting disturbance in a medium that doesn't result in net particle movement. Examples include changes in temperature, pressure, electric or magnetic intensity, electric potential, and elastic deformation.

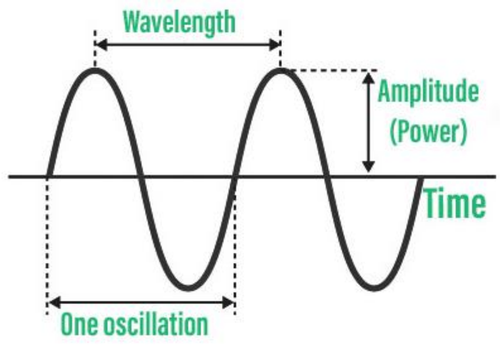


Figure 12.1- Graphical description of waves

12.3 Properties of waves

Waves have following properties-

1. Although the particles of the medium that a wave travels through vibrate slightly around their mean positions, they are not permanently moved in the direction of the wave.
2. Every subsequent particle in the medium moves in a manner that is exactly the same as that of its predecessors, either parallel to or perpendicular to the wave's path of travel.
3. During wave motion, only energy is transferred, but not a piece of the medium.

Types of Waves

The several forms of waves are listed here:

1. Transverse Waves:

Waves in which the medium moves at an angle to the wave's direction.

Examples of transverse waves:

- *Water waves (ripples of gravity waves, not sound through water)*
- *Light waves*
- *S-wave earthquake waves*
- *Stringed instruments*
- *Torsion wave*

A crest is the highest point of a transverse wave. It's a trough at the bottom.

2. Longitudinal Wave:

The movement of the particles in the medium in a longitudinal wave is in the same dimension as the wave's movement direction.

Examples of longitudinal waves:

- *Sound waves*
- *P-type earthquake waves*
- *Compression wave*

Parts of longitudinal waves:

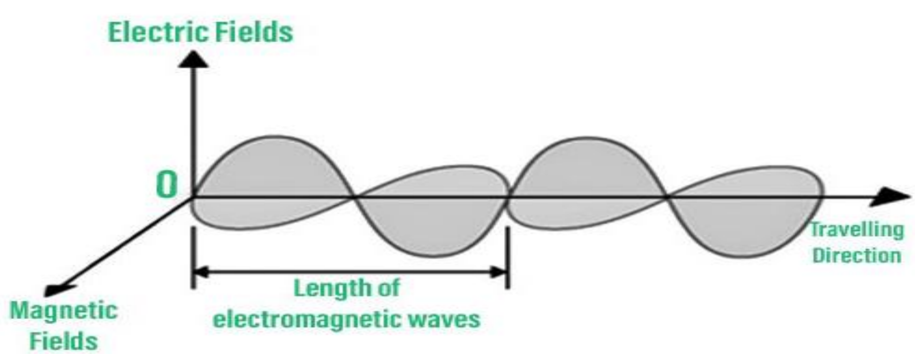
1. *Compression*-The particles are close together in this case.
2. *Rarefaction*-Where the particles are dispersed

3. Electromagnetic Waves:

These are waves that are produced and propagated without the use of a material medium, i.e., they can pass through vacuum and any other material medium.

Examples of electromagnetic waves:

- *visible light*
- *ultra-violet light*
- *radio waves*
- *microwaves*



4. Mechanical waves:

Only a material medium can produce or propagate mechanical waves. Newton's equations of motion apply to these waves.

Examples of mechanical waves:

- *waves on water surface*
- *waves on strings*
- *sound waves*

Mechanical waves are of two types:

1. **Transverse wave motion-** The particles of the medium vibrate at right angles to the wave's propagation direction in transverse waves. Transverse waves include string waves, surface water waves, and electromagnetic waves. The disturbance that travels in electromagnetic waves (which include light waves) is caused by the oscillation of electric and magnetic fields at right angles to the wave's travel direction.
2. **Longitudinal wave motion-** Particles in the medium vibrate back and forth around their mean location along the energy propagation direction in these sorts of waves. They're also known as pressure waves. Longitudinal mechanical waves are what sound waves are.

5. Matter waves:

These waves are linked to the movement of matter particles.

Examples of matter waves:

- *electrons*
- *protons*
- *neutrons*

12.4 Speed formula for wave-

It is the total distance a wave covers in a specific period of time. The following formula can be used to determine the wave speed:

$$\textbf{Wave Speed} = \frac{Distance covered by the wave}{Time taken by the wave}$$

Properties of Waves

The following are the primary characteristics of waves:

- **Amplitude** – A wave is a form of energy transmission. The amplitude of a wave is its height, which is commonly measured in meters. It is proportional to the quantity of energy transported by a wave.
- **Wavelength** – A wavelength is a distance between identical locations in adjacent cycles of crests of a wave. In addition, it is measured in meters.
- **Period** – A wave's period is the amount of time it takes a particle on a medium to complete one complete vibrational cycle. Because the period is a unit of time, it is measured in seconds or minutes.
- **Frequency** – The number of waves passing a spot in a certain amount of time is referred to as the frequency of a wave. The hertz (Hz) unit of frequency measures one wave every second.

> *The frequency's reciprocal is the period, and vice versa.*
>
> ***Period=1 / Frequency***
>
> *OR*
>
> ***Frequency = 1 / Period***

- **Speed** – The speed of an object refers to how quickly it moves and is usually stated as the distance traveled divided by the time it takes to travel. The distance traveled by a specific point on the wave (crest) in a given amount of time is referred to as the wave's speed. A wave's speed is thus measured in meters per second or m/s.

12.5 Wave Behavior

Waves exhibit several interesting behaviors when they interact with their environment or other waves.

12.6 Here are some of the common wave behaviors:

Reflection: When a wave encounters a barrier, it bounces back in the opposite direction. The angle of incidence equals to the angle of reflection. You can see reflection in action when sound waves bounce off a wall and you hear an echo, or when light waves bounce off a mirror and create a reflection.

Refraction: When a wave travels from one medium to another where its speed is different, it bends. This bending is called refraction. The amount of bending depends on the difference in speed between the two mediums.

Refraction is why a straw appears bent when inserted in a glass of water, and why light bends as it enters the atmosphere from space, causing objects to appear slightly displaced.

Diffraction: When a wave encounters an opening or a small obstacle, it bends around the edges and spreads out. Diffraction is why sound waves can bend around corners, and why light waves can spread out from a narrow slit.

Interference: When two waves meet, their crests and troughs can interact with each other, producing either constructive or destructive interference. Constructive interference, when crests of two waves line up, creates wave with larger amplitude. Destructive interference, crest of one wave lines up with the trough of another wave, canceling out the waves.

12.7 Doppler Effect

The variation in a wave's frequency or wavelength in relation to an observer moving with respect to the wave source is commonly referred as Doppler Effect.

It occurs for any type of wave, but it is commonly observed with sound waves.

Source approaching: Waves compressed, higher perceived frequency (e.g., ambulance siren).

Source moving away: Waves stretched, lower perceived frequency (e.g., ambulance siren moving away). • Applies to all waves:

Sound waves (common experience).

12.8 Light waves (astronomical study of stars and galaxies).

Standing Wave

- Instead of propagating, the energy of these waves becomes confined within a specific region, forming a stationary pattern of oscillation.
- Standing waves have points of minimal displacement called nodes and points of maximal displacement called antinodes.
- They are observed in various physical systems, including vibrating strings, acoustic resonance in pipes, and electromagnetic waves in transmission lines.
- Standing waves play a crucial role in phenomena such as musical instrument vibrations, sound resonance, and the behavior of electromagnetic fields in antennas.

13 Wave equation and its general solution

Without actual substance transfer, waves are the symmetrical conveyance of disturbances across a medium. It is not without energy. Simple waves are those that periodically produce ripples at a particular wavelength and frequency. Light and other electromagnetic waves can propagate in a vacuum without the need for a medium. Being mechanical waves, sound waves require a medium to travel through, like air or water. This explains why there is a vacuum in space and why astronauts are unable to hear one another. By obtaining a mathematical expression for the wave equation, some scientists attempted to provide an explanation for this idea from a different perspective. The wave equation has numerous uses in daily life.

12.9 What is a wave equation?

- It is a second-order linear partial differential equation which shows how an oscillation propagates with a certain quantity at a given speed.
- Let's think about how a vibrating string moves. The acceleration at any location on the string is precisely proportional to the string's curvature and points perpendicular to the string, according to the wave equation.
- It develops in domains like as electromagnetism, fluid dynamics, and acoustics.
- There are several kinds of waves, such as mechanical, electromagnetic, and matter waves.
- The wave equation was first discovered by scientists Brook Taylor, who applied the ideas of Newton's second rule of motion.
- A common illustration of a hyperbolic differential equation is the wave equation
- It is important for the fields of plasma physics, general relativity, quantum mechanics, and geophysics.
- The energy carried by a particular wave's oscillation per unit of time is known as the wave's power.
- The ideas of fluid dynamics, optics, gravitational physics, and electromagnetic all depend on the solutions to wave equations.
- The wave equation includes several crucial components, including the wave's structure, frequency, period, speed, amplitude, and energy carried.
- In actuality, the Schrödinger wave equation is a mathematical derivation that takes into account the electron's matter wave nature inside the atom to estimate the position and energy of the electrons in time and space.

12.10 Derivation of wave equation

By selecting just a infinitesimal section of a string, we apply Newton's law to an elastic string in this derivation.

The symbols have there usual meanings:

u(x,t) , t, θ (x,t), T(x,t) and ρ(x) are vertical displacement of string, time, Angle between the horizontal line and the string, Tension, Mass density of the string at position x and time t respectively

The forces applying on the small part of the string are:

(a) In the right direction tension T is acting, its magnitude is T(x+Δx,t), occurs at an angle θ (x+Δx,t) above the horizontal

(b) Tension along the left direction, its magnitude T(x,t) occurs at an angle θ (x,t) which is below the horizontal line.

(c) So many external forces, like gravity. Every external force act vertically represented by F(x,t)Δx.

The mass of the small part of the string is represented by $\rho(x)\sqrt{\Delta x^2 + \Delta u^2}$. As we know, according to the vertical component of Newton's law,

$$\rho(x)\sqrt{\Delta x^2 + \Delta u^2}\,\frac{\partial^2 u}{\partial t^2}(x,t) = T(x + \Delta x, t)\sin\theta(x + \Delta x, t) - T(x,t)\sin\theta(x,t) + F(x,t)\Delta x$$

By taking the limit as $\Delta x \to 0$ and dividing by Δx, we get

$$\rho(x)\sqrt{1 + \left(\frac{\partial u}{\partial x}\right)^2}\,\frac{\partial^2 u}{\partial t^2}(x,t) = \frac{\partial}{\partial x}[T(x,t)\sin\theta(x,t)] + F(x,t) = \frac{\partial T}{\partial x}(x,t)\sin\theta(x,t) + T(x,t)\cos\theta(x,t)\frac{\partial\theta}{\partial x}(x,t) + F(x,t)$$

The above equation is Equation (1) -----

Now we will discard of all the θ s, and the equation will come as,

$$tan\theta(x,t) = \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x} = \frac{\partial u}{\partial x}(x,t)$$

which, further will be

$$\sin sin\, \theta(x,t) = \frac{\frac{\partial u}{\partial x}(x,t)}{\sqrt{1+\left(\frac{\partial u}{\partial x}(x,t)\right)^2}}\ cos\theta(x,t) = \frac{1}{\sqrt{1+\left(\frac{\partial u}{\partial x}(x,t)\right)^2}}\ \ \theta(x,t) = \frac{\partial u}{\partial x}(x,t)\ \ \frac{\partial \theta}{\partial x}(x,t) = \frac{\frac{\partial^2 u}{\partial x^2}(x,t)}{1+\left(\frac{\partial u}{\partial x}(x,t)\right)^2}$$

Now we will substitute these formulae into equation (1). We can simplify it by taking only small vibrations into consideration. Small vibrations means that $|\theta(x,t)| \ll 1$ for all x and t. This will imply that $|tan\theta(x,t)| \ll 1$, therefore $\left|\frac{\partial u}{\partial x}(x,t)\right| \ll 1$ and thus,

$$\sqrt{1 + \left(\frac{\partial u}{\partial x}\right)^2} \approx 1\ sin\theta(x,t) \approx \frac{\partial u}{\partial x}(x,t)\ cos\theta(x,t) \approx 1\ \frac{\partial \theta}{\partial x}(x,t) \approx \frac{\partial^2 u}{\partial x^2}(x,t)$$

The equation above is Equation (2) --------

Now we will substitute these into equation (1), which further gives:

$$\rho(x)\frac{\partial^2 u}{\partial t^2}(x,t) = \frac{\partial T}{\partial x}(x,t)\frac{\partial u}{\partial x}(x,t) + T(x,t)\frac{\partial^2 u}{\partial x^2}(x,t) + F(x,t)$$

This equation is Equation (3) ----------

Two unknown values are present in in the above equation, which are (u and T).

Now we use of the second equation that is horizontal component of Newton's law. For assume for conveyance that there is only transverse vibrations exists. The small part of the string will move only in the vertical direction. Therefore, net force in the horizontal direction is zero. Hence,

$$T(x + \Delta x, t)cos\theta(x + \Delta x, t) - T(x,t)cos\theta(x,t) = 0$$

Now we will take the limit as Δx, which tends to zero, and divide by Δx, which further gives us:

$$\frac{\partial}{\partial x}[T(x,t)cos\theta(x,t)] = 0$$

If we consider small amplitude vibrations, $cos\theta$ is quite near to 1, and $\frac{\partial T}{\partial x}(x,t)$ is also near to 0. We can see that T is a function of t only, which can be measured by the force we are applying on the ends of the string, at time t. Thus, the equation formed will be,

$$\rho(x)\frac{\partial^2 u}{\partial t^2}(x,t) = T(t)\frac{\partial^2 u}{\partial x^2}(x,t) + F(x,t)$$

This is Equation number (4).

Here, the string density is constant, not depends on position x. string tension constant T(t), is also independent of t. Furthermore, there exist zero external forces F, so we can end with the below mentioned equation:

$$\frac{\partial^2 u}{\partial t^2}(x,t) = c^2 \frac{\partial^2 u}{\partial x^2}(x,t)$$

Here,

$$c = \sqrt{\frac{T}{\rho}}$$

12.11 Applications of wave equations

1. The wave equation is applied to many different phenomena, including gravitational waves, light waves, sound waves, and string theory.
2. It facilitates our understanding of the motion of strings and fluid surfaces like waves in water.
3. Through the use of interference to superimpose waves, we can obtain information about them through the process of interferometers.
4. This technique is applied in the measuring of surface imperfections, tiny displacements, and changes in refractive index.
5. In 3D spaces, the probability distribution is found using the wave function.
6. The basis of wave mechanics and a tool for understanding the atomic structures of different elements is the Schrödinger wave equation. It also demonstrates how matter has wave-like characteristics.
7. Applications for the wave theory can be found in many domains, including wireless communications, musical instruments, and speeding car detection.

12.12 Solution of the wave equation

Let $y = X(x)\,T(t)$ is the solution of equation (1), where X is a function of (x) only and T is a function of (t) only.

Solution of the wave equation

The wave equation is

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2} \quad \text{-----------(1)} .$$

Let y = X(x) .T(t) be the solution of (1), where „X‟ is a function of „x‟ only and „T‟ is a function of „t‟ only.

Then $\frac{\partial^2 y}{\partial t^2} = X\,T''$ and $\frac{\partial^2 y}{\partial x^2} = X''\,T.$

Substituting these in (1), we get

$$X\,T'' = a^2 X''\,T.$$

$$\text{i.e,} \quad \frac{X''}{X} = \frac{T''}{a^2 T} \quad \text{---------------(2).}$$

Of these three solutions, we have to select that particular solution which suits the physical nature of the problem and the given boundary conditions. Since we are dealing with problems on vibrations of strings, „y‟ must be a periodic function of „x‟ and „t‟.

$$\text{Therefore,} \qquad \frac{X''}{X} = \frac{T''}{a^2T} = k \text{ (say).}$$

Hence, we get $X'' - kX = 0$ and $T'' - a^2 kT = 0$. --------------(3).

Solving equations (3), we get

(i) when 'k' is positive and $k = \lambda^2$, say

$$X = c_1 e^{\lambda x} + c_2 e^{-\lambda x}$$
$$T = c_3 e^{a\lambda t} + c_4 e^{-a\lambda t}$$

(ii) when 'k' is negative and $k = -\lambda^2$, say

$$X = c_5 \cos\lambda x + c_6 \sin\lambda x$$
$$T = c_7 \cos a\lambda t + c_8 \sin a\lambda t$$

(iii) when 'k' is zero.

$$X = c_9 x + c_{10}$$
$$T = c_{11} t + c_{12}$$

Thus the various possible solutions of the wave equation are

$$y = (c_1 e^{\lambda x} + c_2 e^{-\lambda x})(c_3 e^{a\lambda t} + c_4 e^{-a\lambda t}) \quad \text{------------(4)}$$
$$y = (c_5 \cos\lambda x + c_6 \sin\lambda x)(c_7 \cos a\lambda t + c_8 \sin a\lambda t) \quad \text{-----------(5)}$$
$$y = (c_9 x + c_{10})(c_{11} t + c_{12}) \quad \text{------------(6)}$$

Now the left side of (2) is a function of „x‟ only and the right side is a function of „t‟ only. Since x and t are independent variables, equation (2)satisfied only if each side is equal to a constant value. Solution must involve trigonometric terms. so, the solution (5) is given by,

i.e, $y = (c_5 \cos \lambda x + c_6 \sin \lambda x)(c_7 \cos a\lambda t + c_8 \sin a\lambda t)$ is the only suitable solution of the wave equation.

Summary

By understanding the behavior described by the wave equation and its solutions, engineers can predict how vibrations will propagate through different materials and structures, allowing them to optimize designs for various applications and mitigate potential issues related to resonance or structural failure. When a bar is subjected to a longitudinal force or displacement, it can set up waves that propagate along its length. These waves cause particles within the material to oscillate back and forth along the direction of wave propagation.

Keywords

Structural dynamics, Forced vibration, Torsional vibration

Objective type question

1. Find the natural frequency in Hz of the free longitudinal vibrations if the displacement is 2mm.

   a) 11.14

   b) 12.38

   c) 11.43

   d) 11.34

2. If the spring displacement is high then the frequency of the spring increases.

   a) True

   b) False

3. Find the displacement in mm of the free longitudinal vibrations if the Natural frequency is 15 Hz.

   a) 1.1

   b) 1.2

   c) 1.5

   d) 1.6

4. Find the displacement in mm of the free longitudinal vibrations if the Natural frequency is 20 Hz.

   a) 0.1

   b) 0.2

   c) 0.5

   d) 0.6

5. Which of the following methods will give an incorrect relation of the frequency for free vibration?

   a) Equilibrium method

   b) Energy method

c) Reyleigh’s method

d) Klein’s method.

Self assessment

1. Explain the Longitudinal vibrations in bars
2. How do vibrations occur in bars?"
3. What is the formula to calculate the fundamental frequency of longitudinal vibrations in a bar
4. How does the wave equation describe longitudinal vibrations in bars, and what is its general solution?

Chapter 13

Study of Vibrations in Bars in Different Cases

Objectives

1. Studying vibrations in bars under different boundary conditions
2. Investigate how various constraints affect the modes of vibration and natural frequencies of the system.
3. Understanding these different cases helps engineers and physicists design and analyze structures more effectively, ensuring they can predict and control the behavior of vibrations in real-world applications.

13.1 Vibration in Uniform Bar

Let us consider a prismatic bar of length L subjected to longitudinal vibrations as shown in figure 13.1

The cross sectional area (A) of the bar and young's modulus (E) of the material of the bar, ρ be the density of the material and m be the mass per unit length.

Let us assume that the bar should be thin and of uniform cross section throughout of its length and subjected to axial force F and there will be displacement u along the rod that will be a function of both position x and time t, because the rod has an infinite number of natural modes of vibrations.

The distribution of the displacement will differ with each mode as shown in 13.1 (a)

Let us consider a small elemental length dx at a distance x from the left end and F be the axial force on a small elemental length. The force on the other side that is right side of small elemental length is equal to $\left(F + \frac{\partial F}{\partial x} dx\right)$

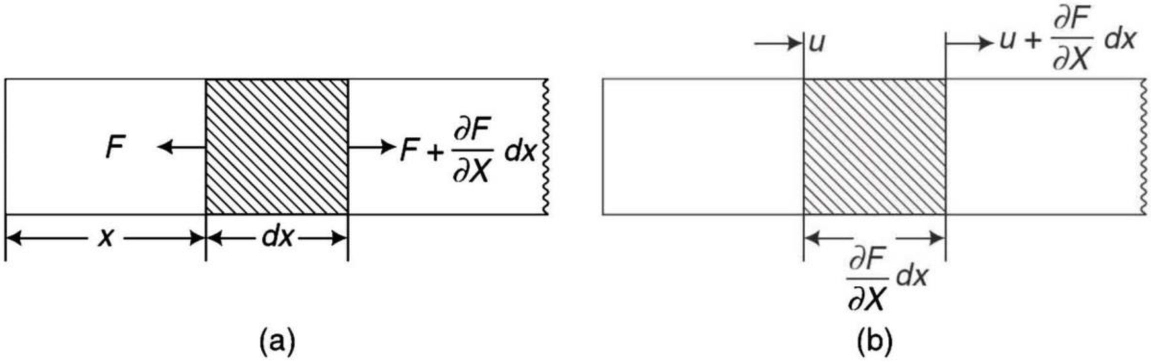


Figure 13.1 Longitudinal vibration of Rod

If u is the displacement at a distance x from the left side and $\left(u + \frac{\partial u}{\partial x}dx\right)$ displacement at a distance x+dx at the right side of small elemental length. Now it is clear from the figure, due to these axial forces on the small elemental length dx there is a change length by an amount equal to $\left(u + \frac{\partial u}{\partial x}dx - u\right) = \left(\frac{\partial u}{\partial x}dx\right)$.

We know from mechanics of the material, when an element or a body is subjected either to the tension or compression, it undergoes stress, strain and deformation.

By the definition of strain (ε) = change in length/ original length

$$\epsilon = \frac{\frac{\partial u}{\partial x}dx}{dx} = \frac{\partial u}{\partial x} \qquad (1)$$

13.2 Net Force Acting on the Small Element,

$$\left(F + \frac{\partial F}{\partial x}\partial x\right) - F = \text{(Mass)} \times \text{(Acceleration of the element)}$$

$$= dm \times \frac{\partial^2 u}{\partial t^2},$$ Where dm = mass of the small elemental length

$$\frac{\partial F}{\partial x}dx = (\rho dxA)\left(\frac{\partial^2 u}{\partial t^2}\right)$$

ρ = Density and $dx\,A$ = Volume of the small elemental length (2)

We know that definition of stress (s) = Load /Area, σ= F/A or F=σA

$$\frac{\partial F}{\partial x} = \frac{\partial \sigma}{\partial x}A, \left(\frac{\partial F}{\partial x}\right)dx = \left(\frac{\partial \sigma}{\partial x}\right)dx\,A \qquad (3)$$

Equation 2 can be written with the help of above equation as

$$\left(\frac{\partial\sigma}{\partial x}\right) dxA = (\rho dxA)\left(\frac{\partial^2 u}{\partial t^2}\right) \tag{4}$$

According to Hook's low, stress is directly proportional to strain with in elastic limit.

i.e. ∝∈ **,** s = E ∈

$$E = \frac{\sigma}{\in},$$

$$\frac{\text{Stress}}{\text{Strain}} = E, \text{ where } E = \text{Young's modulus}, \sigma = \in E, \left(\frac{\partial\sigma}{\partial x}\right) dxA = \left(\frac{\partial^2 u}{\partial t^2}\right) dxAE \tag{5}$$

With the help of equation 4 and 5, we get

$$\left(\frac{\partial\in}{\partial x}\right) dxAE = (\rho dxA)\left(\frac{\partial^2 u}{\partial t^2}\right)$$

$\left(\in = \frac{\partial u}{dx}\right)$ form equation 1.

$$\left(\frac{E}{\rho}\right)\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) = \left(\frac{\partial^2 u}{\partial t^2}\right), \frac{E}{\rho}\left(\frac{\partial^2 u}{\partial x^2}\right) = \frac{\partial^2 u}{\partial t^2}, \quad a^2\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \text{ where } a^2 = E/\rho$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{a^2}\frac{\partial^2 u}{\partial t^2} \tag{6}$$

This is the wave equation which is identical to $\left(\frac{\partial^2 y}{\partial x^2} = \frac{1}{a^2}\frac{\partial^2 y}{\partial t^2}\right)$.

The general solution will be same as in the case of lateral vibrations. A Solution of the form is as in

u(x,t) = X(x)T(t) so,

$$X(x) = A \sin\frac{Z_n x}{a} + B \cos\frac{Z_n x}{a}, T(t) = C \sin Z_n t + D \cos Z_n t$$

Will result in to the general solution as

$$u(x, t) = \sum_{n=1}^{\infty}\left(A \sin\frac{Z_n}{a}x + B \cos\frac{Z_n}{a}x\right)(C \sin Z_n t + D \cos Z_n t) \tag{7}$$

Case I

Longitudinal vibration for a free-free Beam with zero initial displacement:

The system is as shown in figure

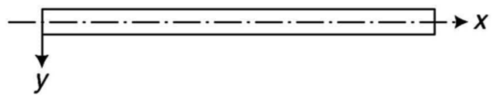


Figure 13.2 Longitudinal vibration of a beam

We know that the general solution of longitudinal vibration of a uniform bar is given by equation (7).

$$u(x, t) = \sum_{n=1}^{\infty} \left(A \sin \frac{Z_n}{a} x + B \cos \frac{Z_n}{a} x\right)(C \sin Z_n t + D \cos Z_n t)$$

Where $a = \sqrt{\frac{E}{\rho}}$ and $Z_n = 2\pi fn$ where Zn is the natural frequency.

The boundary conditions for the above particular system (Free –Free Beam) with zero initial displacement are

$\left(\frac{\partial u}{\partial x}\right)_{x=0} = 0$ and $\left(\frac{\partial u}{\partial x}\right)_{x=L} = 0$ (for free end on both ends, strain is zero)

Differentiating the above equation (7) with respect to x partially and applying these boundary conditions to the general solution we get.

$$\left(\frac{\partial u}{\partial x}\right) = \left(A \frac{Z_n}{a} \cos \frac{Z_n}{a} x - B \frac{Z_n}{a} \sin \frac{Z_n}{a} x\right)(C \sin Z_n t + D \cos Z_n t) \qquad (8)$$

$$\left(\frac{\partial u}{\partial x}\right)_{x=0} = A \frac{Z_n}{a}(C \sin Z_n t + D \cos Z_n t) \quad \therefore A = 0$$

$$\left(\frac{\partial u}{\partial x}\right)_{x=L} = \left(-B \frac{Z_n}{a} \sin \frac{Z_n}{a} x\right)(C \sin Z_n t + D \cos Z_n t),$$

$$\left(-B \frac{Z_n}{a} \sin \frac{Z_n}{a} L\right)(C \sin Z_n t + D \cos Z_n t)$$

By using solution of wave equation $C_n = \frac{2}{L}\int_0^L s(x) \sin \frac{n\pi x}{L} dx \qquad D_n = \frac{2}{Z_n L}\int_0^L v(x) \sin \frac{n\pi x}{L} dx.$

We can determine the values of constants C and D from initial conditions, So

$$\sin \frac{Z_n}{a} L = 0, \sin n\pi, Z_n = \frac{n\pi a}{L}, n = 1, 2, 3...$$

We know that $Z_n = 2\pi f_n,\ 2\pi f_n = 2\pi a / L$ therefore, The natural frequency $f_n = \frac{n}{2L}a$, But $a = \sqrt{\frac{E}{\rho}}$ therefore

$f_n = \frac{n}{2L}\sqrt{\frac{E}{\rho}}$ 'n' represent the order of the mode.

Case II

Longitudinal vibration for a uniform beam of length L one end of which is fixed and either end is free.

We know that the general solution of longitudinal vibration of a uniform bar is given by Equation number (7).

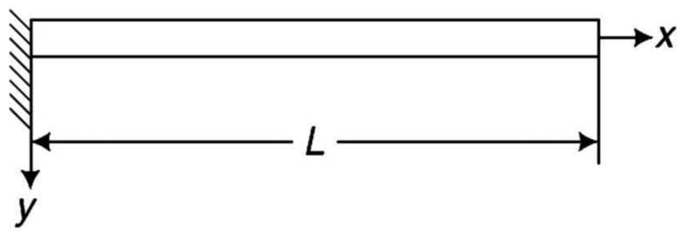


Figure 13.3. Uniform Bar

$$u(x, t) = \sum_{n=1}^{\infty} \left(A \sin \frac{Z_n}{a} x + B \cos \frac{Z_n}{a} x\right) (C \sin Z_n t + D \cos Z_n t)$$

The boundary condition for the above particular system (one end of which is free and one is fixed) are

$(u)_{x=0} = 0$ (displacement is zero at fixed end.) and $\left(\frac{\partial u}{\partial x}\right)_{x=L} = 0$ strain is zero at free end. Differentiate equation (8) with respect to x partially. We get

$$\left(\frac{\partial u}{\partial x}\right) = \left(A \frac{Z_n}{a} \cos \frac{Z_n}{a} x - B \frac{Z_n}{a} \sin \frac{Z_n}{a} x\right) (C \sin Z_n t + D \cos Z_n t) \qquad (9)$$

Appling the boundary condition to the general solution of equation

$$\theta_1 = \frac{\delta y}{\delta x}, \text{ and } \tan \theta_2 = \frac{\delta y}{\delta x} + \frac{\delta}{\delta x}\left(\frac{\delta y}{\delta x}\right)dx,\ \theta_2 = \theta_1 + \frac{\delta \theta_1}{\delta x} dx$$ we have B=0

$$0 = A \frac{Z_n}{a} \cos \frac{Z_n}{a} L (C \sin Z_n t + D \cos Z_n t) \text{ or } \cos \frac{Z_n}{a} L = 0 = \cos \frac{n\pi}{2},$$

Where n= 1,3,5, -----

And

$$A \neq 0.\ \frac{Z_n}{a} L = \frac{n\pi}{2}, \quad Z_n = \frac{n\pi a}{2L} \quad \text{But } Z_n = 2\pi f_n,\ 2\pi f_n = \frac{na\pi}{2L}$$

$$f_n = \frac{n}{4L}\sqrt{\frac{E}{\rho}} \quad \therefore\ a = \sqrt{\frac{E}{\rho}}$$

The general solution of longitudinal vibration of a uniform bar can be written as

$$u(x,\ t) = \sum_{n=1,3,5.}^{\infty} \sin\frac{nx\pi}{2L}\left(C \sin\frac{na\pi}{2L} t + D \cos\frac{na\pi}{2L} t\right).$$

Case –III

L is a length of a bar of uniform cross section fixed at both of its ends

A bar is subjected to longitudinal vibrations having a constant velocity V_0 at all points.

From the general solution of longitudinal vibration of a uniform bar equation (7) can be written as

$$u(x,\ t) = \sum_{n=1,2,3.}^{\infty}\left(A \sin\frac{Z_n}{a} x + B \cos\frac{Z_n}{a} x\right)(C \sin Z_n t + D \cos Z_n t)$$

The boundary condition for the above particular system fixed at both ends are x =0, displacement =0 that is u(0,t) = 0, u(L,t) = 0

By using the first boundary condition in the above general solution of longitudinal vibration of a uniform bar equation (7), we get

$$u(x,\ t) = \sum_{n=1,2,3.}^{\infty} \sin\frac{n\pi x}{L}(C \sin Z_n t + D \cos Z_n t)$$

B = 0

And by using secondary boundary conditions, we have $Z_n L/a = 0 = \sin n\pi$ n = 1,2,3….but

$$f_n = \frac{n\pi a}{L}, a = \sqrt{\frac{E}{\rho}}$$

After substituting the above values equation number (2) becomes

Again the initial conditions are u(x,0) = 0, u(x,t) = V_0

By using the first initial condition in the above general solution. we get

$$0 = \sum_{n=1,2,3,...}^{\infty} \sin\frac{n\pi x}{L} \cdot D,\ D = 0$$

Then the equation is

$$u(x,\ t) = \sum_{n=1,2,3,...}^{\infty} \sin\frac{n\pi x}{L} \cdot C \sin Z_n t$$

By using the II initial condition in the above solution, we get

Then the equation becomes $$\dot{u}(x, t) = \sum_{n=1,2,3,...}^{\infty} \sin\frac{n\pi x}{L} \cdot C Z_n \cos Z_n t$$

$$\dot{u}(x, 0) = \sum_{n=1,2,3,...}^{\infty} C Z_n \sin\frac{n\pi x}{L} = V_0$$

$$C = \frac{2}{n\pi a}\int_0^L V_0 \sin\frac{n\pi x}{L}\, dx \text{ (Eq. 9.18)} \quad C = \frac{2V_0 L}{n^2\pi^2 a}(1 - \cos n\pi)$$

$$C = \frac{4V_0 L}{n^2\pi^2 a} \text{ when } n = 1, 3, 5... \quad \text{and } C = 0 \text{ when } n = 2, 4, 6...$$

Finally the required expression can be written as

$$u(x, t) = \frac{4V_0 L}{\pi^2 a} \sum_{n=1,3,5,...}^{\infty} \frac{1}{n^2} \sin\frac{n\pi x}{L} \sin\frac{n\pi a}{L} t$$

Summary

By studying these different cases of vibration in bars, engineers can gain insights into how boundary conditions influence the behavior of waves and vibrations in structural elements. This knowledge is crucial for optimizing designs, ensuring structural integrity, and minimizing unwanted vibrations in various engineering applications.

Keywords

Longitudinal waves, Mode shapes, Travelling waves , Structural dynamics

Objective type questions

1. Static deflection and frequency are independent of each other.

   a) True

   b) False

2. What is the ultimate tensile strength of a material?

   a) The maximum stress a material can withstand without breaking

   b) The stress at which a material starts to deform permanently

   c) The stress at which a material reaches its elastic limit

   d) The stress at which a material is completely compressed

3. What is the formula for calculating stress?

a) Stress = Force × Distance

b) Stress = Force / Area

c) Stress = Mass × Acceleration

d) Stress = Work / Time

4. When a material experiences stress, what does it induce?

a) Strain

b) Displacement

c) Velocity

d) Acceleration

5. What is stress?

a) Force per unit area

b) Force acting on a material

c) Change in shape due to external force

d) None of the above

Self Assessment

1. What are some different cases in the study of vibrations in bars,
2. How do they affect the behavior of the vibrations?
3. What is a longitudinal vibration
4. What is the longitudinal vibration of a string
5. What is the formula for frequency of longitudinal vibration?

Chapter 14
Transverse Vibrations in a Bar

Objectives

1. Studying transverse vibrations in a bar
2. Understand how waves propagate perpendicular to the length of the bar, resulting in bending or flexural vibrations.
3. Exploring the wave equation governing transverse vibrations and considering different boundary conditions, engineers and physicists can predict the behavior of vibrations in structures like beams, rods, and tuning forks, enabling the design and optimization of these systems for various applications.

14.1 Wave Equation & its General Solution

Without actual substance transfer, waves are the symmetrical conveyance of disturbances across a medium. It is not without energy. Simple waves are those that periodically produce ripples at a particular wavelength and frequency. In vacuum, Light and electromagnetic waves can propagate. Being mechanical waves, sound waves require a medium to travel through, like air or water. This explains why there is a vacuum in space and why astronauts are unable to hear one another. By obtaining a mathematical expression for the wave equation, some scientists attempted to provide an explanation for this idea from a different perspective. The wave equation has numerous uses in daily life.

14.2 What is a Wave Equation?

- It is a second-order linear partial differential equation that shows how an oscillation propagates with a certain quantity at a given speed.
- Let's think about how a vibrating string moves. The acceleration at any location on the string is precisely proportional to the string's curvature and points perpendicular to the string, according to the wave equation.
- It develops in domains like as electromagnetism, fluid dynamics, and acoustics.
- There are several kinds of waves, such as mechanical, electromagnetic, and matter waves.
- The wave equation was first discovered by scientists Brook Taylor, who applied the ideas of Newton's second rule of motion.
- One-dimensional wave equation discovered in 1746 by d'Alembert

- Ten years after d'Alembert, Euler discovered the three-dimensional wave equation.
- A common illustration of a hyperbolic differential equation is the wave equation
- It is important for the fields of plasma physics, general relativity, quantum mechanics, and geophysics.
- The energy carried by a particular wave's oscillation per unit of time is known as the wave's power.
- The ideas of fluid dynamics, optics, gravitational physics, and electromagnetic all depend on the solutions to wave equations.
- By the application of second law of motion (F=ma) to a tiny portion or infinitesimal length (dx) of the string yields the wave equation.
- The wave equation includes several crucial components, including the wave's structure, frequency, period, speed, amplitude, and energy carried.
- In actuality, the Schrödinger wave equation is a mathematical derivation that takes into account the electron's matter wave nature inside the atom to estimate the position and energy of the electrons in time and space.

14.3 Derivation of Wave Equation

By selecting just a small section of the string, we will apply Newton's law to an elastic string in this derivation.

Here:

$u(x,t)$ = Vertical displacement of string in x axis, t is time (x,t) = is the angle between the horizontal line and the string at a position x and time t

$T(x,t) = T$, tension in the element of the string at x and time t

$\rho(x) = \rho$, mass density at position x

The forces applying on the small part of the string are:

(a) Tension at an angle $\theta\ (x+\Delta x,t)$ above the horizontal, acting in a rightward direction and with a magnitude of $T(x+\Delta x,t)$.

(b) In the left direction, Tension act, which has its magnitude $T(x,t)$ and occurs at an angle $\theta\ (x,t)$ below the horizontal

(c) In addition, there are other external forces like gravity. Every external force, indicated by $F(x,t)\Delta x$, will be expected to act vertically.

The mass of the small part of the string is represented by $\rho(x)\sqrt{\Delta x^2 + \Delta u^2}$. As we know, according to the vertical component of Newton's law,

$$\rho(x)\sqrt{\Delta x^2 + \Delta u^2}\frac{\partial^2 u}{\partial t^2}(x,t) = T(x + \Delta x, t)sin\theta(x + \Delta x, t) - T(x,t)sin\theta(x,t) + F(x,t)\Delta x$$

By taking the limit as $\Delta x \to 0$ and dividing by Δx, we get

$$\rho(x)\sqrt{1 + \left(\frac{\partial u}{\partial x}\right)^2}\frac{\partial^2 u}{\partial t^2}(x,t) = \frac{\partial}{\partial x}[T(x,t)sin\theta(x,t)] + F(x,t) = \frac{\partial T}{\partial x}(x,t)sin\theta(x,t) + T(x,t)cos\theta(x,t)\frac{\partial\theta}{\partial x}(x,t) + F(x,t)$$

The above equation is Equation (1) -----

Now we will discard of all the θ s, and the equation will come as,

$$tan\theta(x,t) = \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x} = \frac{\partial u}{\partial x}(x,t)$$

which, further will be

$$\sin sin\,\theta(x,t) = \frac{\frac{\partial u}{\partial x}(x,t)}{\sqrt{1+\left(\frac{\partial u}{\partial x}(x,t)\right)^2}} \quad cos\theta(x,t) = \frac{1}{\sqrt{1+\left(\frac{\partial u}{\partial x}(x,t)\right)^2}} \quad \theta(x,t) = \frac{\partial u}{\partial x}(x,t) \quad \frac{\partial\theta}{\partial x}(x,t) = \frac{\frac{\partial^2 u}{\partial x^2}(x,t)}{1+\left(\frac{\partial u}{\partial x}(x,t)\right)^2}$$

Now we will substitute these formulae into equation (1). We can simplify it by taking only small vibrations into consideration. Small vibrations means that $|\theta(x,t)| \ll 1$ for all x and t. This will imply that $|tan\theta(x,t)| \ll 1$, therefore $\left|\frac{\partial u}{\partial x}(x,t)\right| \ll 1$ and thus,

$$\sqrt{1 + \left(\frac{\partial u}{\partial x}\right)^2} \approx 1 \; sin\theta(x,t) \approx \frac{\partial u}{\partial x}(x,t) \; cos\theta(x,t) \approx 1 \; \frac{\partial\theta}{\partial x}(x,t) \approx \frac{\partial^2 u}{\partial x^2}(x,t)$$

The equation above is Equation (2) --------

Now we will substitute these into equation (1), which further gives:

$$\rho(x)\frac{\partial^2 u}{\partial t^2}(x,t) = \frac{\partial T}{\partial x}(x,t)\frac{\partial u}{\partial x}(x,t) + T(x,t)\frac{\partial^2 u}{\partial x^2}(x,t) + F(x,t)$$

This equation is Equation (3) ----------

Here u and T are unknown in the above equation.

By using second equation, that is horizontal component of Newton's law of motion. For explanation, we consider that only transverse vibrations exist. In vertical direction small part of the string will move. Therefore, the horizontal net force on it will be zero. Hence,

$$T(x + \Delta x, t)cos\theta(x + \Delta x, t) - T(x, t)cos\theta(x, t) = 0$$

Now we will take the limit as Δx, which tends to zero, and divide by Δx, which further gives us:

$$\frac{\partial}{\partial x}[T(x, t)cos\theta(x, t)] = 0$$

If we consider small amplitude vibrations, $cos\theta$ is quite near to 1, and $\frac{\partial T}{\partial x}(x, t)$ is also near to 0. We can see that T is a function of t only, which can be measured by the force we are applying on the ends of the string, at time t. Thus, the equation formed will be,

$$\rho(x)\frac{\partial^2 u}{\partial t^2}(x, t) = T(t)\frac{\partial^2 u}{\partial x^2}(x, t) + F(x, t)$$

This equation is Equation (4).

Here, the string density is constant, its value does not dependent on x, string tension constant T(t),time independent. Moreover, no external force exists, so we can conclude with the following equation:

$$\frac{\partial^2 u}{\partial t^2}(x, t) = c^2\frac{\partial^2 u}{\partial x^2}(x, t)$$

Here,

$$c = \sqrt{\frac{T}{\rho}}$$

14.4 Applications of Wave Equations

1. The wave equation is applied to study different phenomena like gravitational, electromagnetic, sound wave and string theory.
2. To determine the motion of a strings and fluid surfaces (water waves).
3. Through the use of interference to superimpose waves, we can obtain information about them through the process of interferometers.
4. This technique is applied in the measuring of surface imperfections, tiny displacements, and changes in refractive index.
5. In 3D spaces, wave function is founds probability distribution.
6. The basis of wave mechanics and a tool for understanding the atomic structures of different elements is the Schrödinger wave equation. It also demonstrates how matter has wave-like characteristics.

7. Applications for the wave theory can be found in many domains, including wireless communications, musical instruments, and speeding car detection.

14.5 At the End of the String, Boundary Conditions:

Adding Opposite Pulses

To start working with waves, we first moved the end of a string (or spring) to create a pulse that we could see traveling along without any discernible form change. We demonstrated that our observation could be mathematically expressed: assuming that the string was initially at rest along the x-axis, we could clearly see that a function of the kind $y=f(x-vt)$ characterized its displacement y at location x at time t. This function maintains its shape but moves at a speed of v to the right as time goes on.

Next, we used Laws of Motion to examine the dynamics of the vibrating thread on a small sample of string. This shows an equation that all string vibrations must follow: the wave equation. The Equation satisfied by our observed form for the moving pulse, $y = f(x - vt)$, which is assuring.

One extremely significant feature of the wave equation is that the addition of two solutions of a given wave function is also becomes a solution. The overall displacement of the rope will therefore be equal to the sum of the displacements corresponding to each individual pulse. For example, if you and a friend transmit pulses down a rope from the other end, the pulses will pass directly through each other and then overlap. As we'll see, this provides a crucial hint for comprehending what occurs when a pulse hits the string's end.

Reflection of Pulse

When pulses reached what happened at the string's ends. There are two possible outcomes:

(a) The string's end is fixed, or

(b) End of the string is move up and down freely

These are known as the free end and fixed end boundary conditions. Since the thread must be under tension in order for the wave to propagate at all, you may be wondering how the string could possibly have a loose end. This is set up so that the string ends on a ring that may freely move up and down a smooth rod that is perpendicular to the string's direction.

14.6 An experiment into Free and Fixed End Reflection

We present an example where a wire is under strain and thin, perpendicular parallel rods are fastened to it at their centers. Waves will move over this array slowly enough to be easily followed, and white color is paint at the end of these rods for visibility. Sending a pulse down from one end, holding it fixed or allowing it to move freely, and then watching what occurs when the pulse reaches the other end are simple steps to do.

Figure 14.1

It is discovered that the pulse, after it reaches the fixed end, is reflected in the same shape but with a different sign. For example, if the pulse caused the string to bulge in the +y direction prior to reflection, it will now swell in the −y direction following reflection as shown in figure 14.1. On the other hand, the pulse is reflected without changing sign if the end rod is free to rotate.

14.7 Evaluating Changes in Sign in Pulse Reflection

Doing a new experiment and sending two pulses down a rope from opposite ends and carefully observing as they pass in the middle can help you understand what happens when a pulse is reflected. Let's begin with two pulses that have the same shape but different signs. Using a spreadsheet, we will create the pulses and track them as they go by. Recall that the string's total displacement at any given time is the result of adding the displacements of each individual pulse. The two distinct pulses appear as follows:

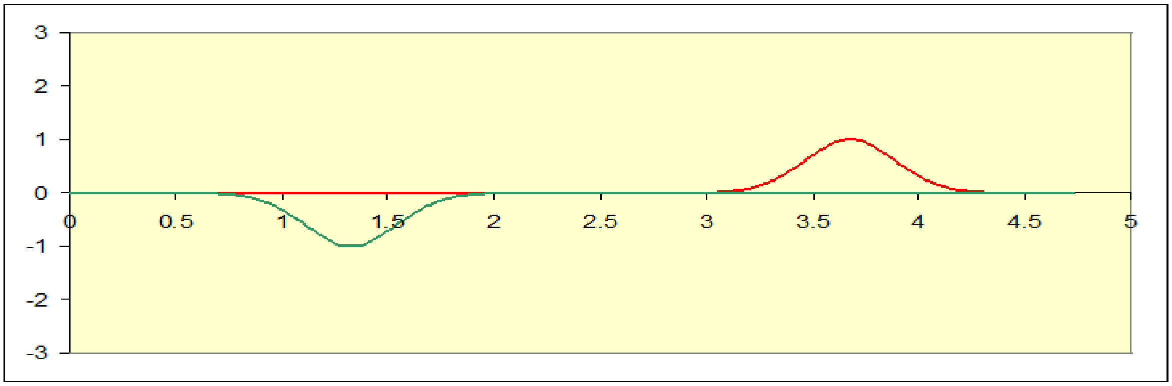


Figure 14.2

Keeping in mind that the red and green lines together conceal the green along the axis, that the red and green are independent solutions to the wave equation, and that the total of the two pulses is also a solution, which is represented by the black line in the diagram below, which represents the string's actual position at a certain point after the pulses are sent on their way:

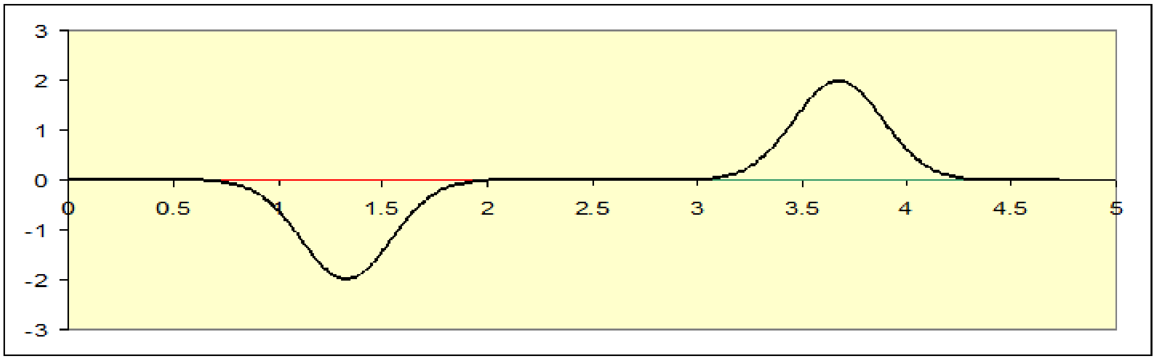


Figure 14.3

As time passes and the two pulses are tracked, they come together:

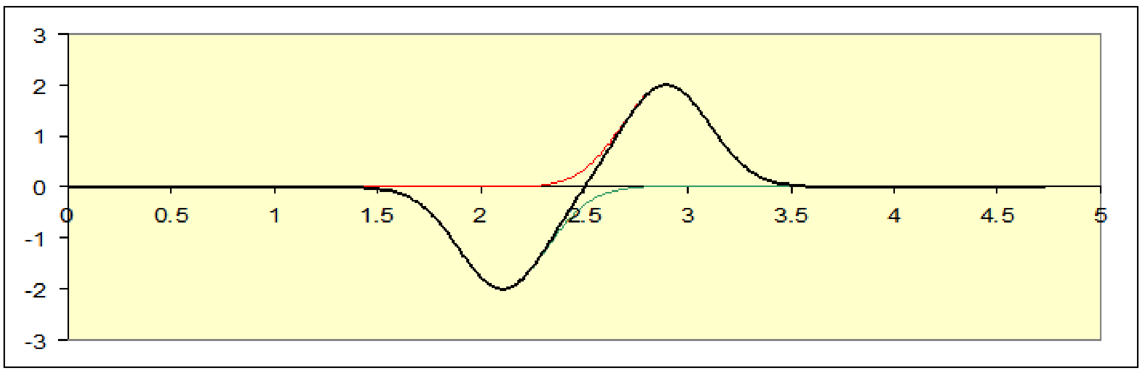


Figure 14.4

The red pulse is now moving to the left, and the green pulse is moving to the right:

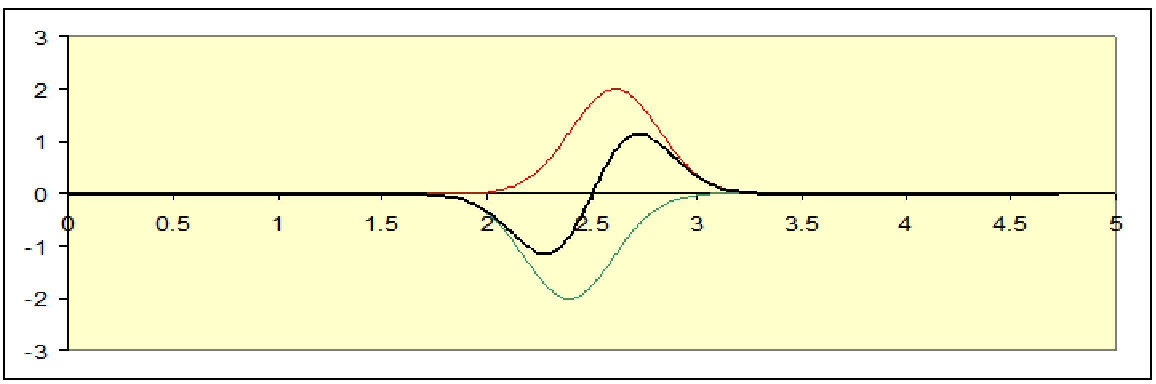


Figure 14.5

They move on (observe the string):

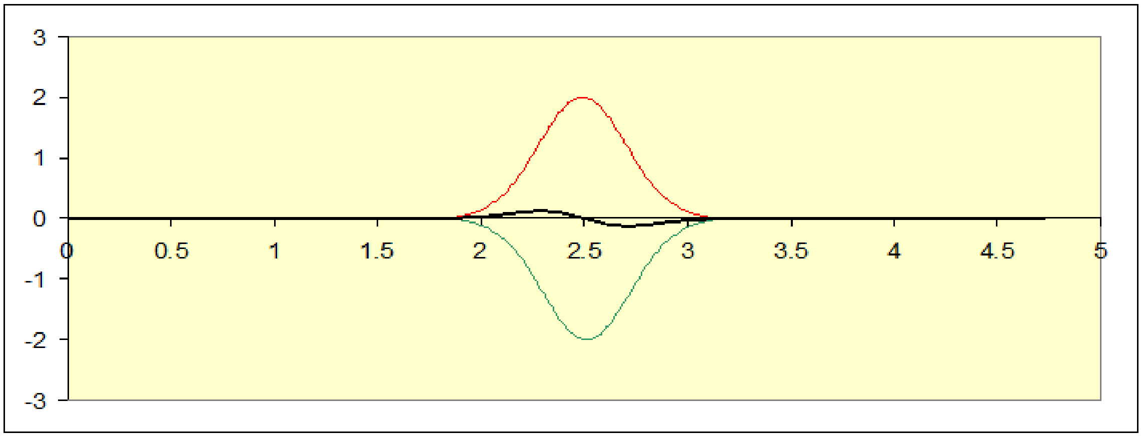


Figure 14.6

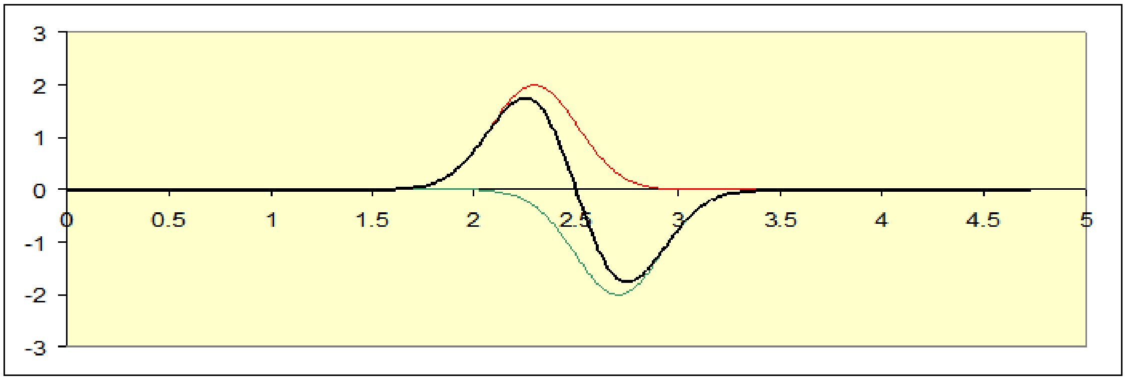


Figure 14.7

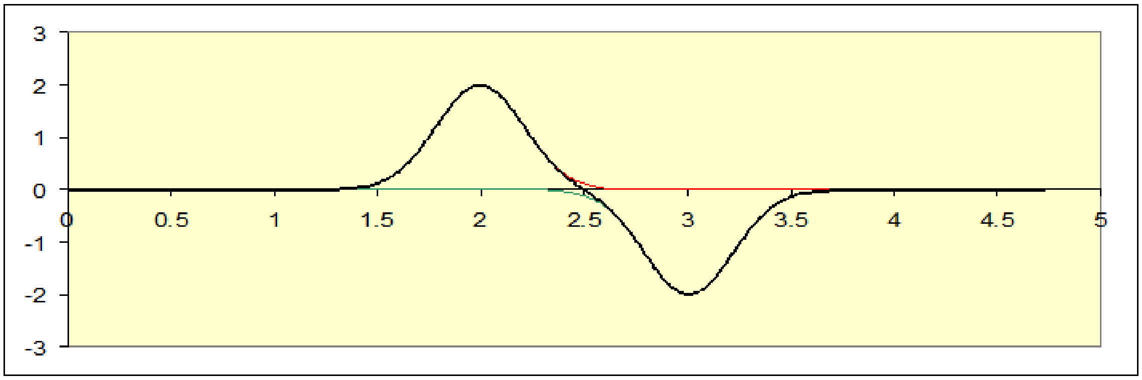


Figure 14.8

Examining this series of images again, you will see that the string at x = 2.5, the center, never moves. It could as well have been fastened with nails. Assume that after nailing it in place, we only transmitted a single pulse down from the left end and cut the string to the right of the center. Solve equation of a wave in a string subjected to the fixed (right) end on the right and a pulse being delivered down from the left end in order to observe what would happen. However, the solution to the wave equation for the entire string involving the first two pulses mentioned above is already known, and the midpoint was constant throughout.

This answer, limited to the left side, must be the same solution because it solves the same equation with the same boundary condition and starting configuration as the two pulses on the entire string scenario! It forces us to conclude that the reflected pulse curves upward when a downward-curving pulse is directed towards a fixed end; it simply follows the same pattern as the left-hand half in our two-pulse full string solution. It goes without saying that this is what our experimental results show.

14.8 Condition of Free End Boundary

If we assume a bar model, in which transmit a pulse down from the left, but we rotate the bar freely on the right-hand end rather than fixing it. What takes place? Remember that the accelerating force on the string depended on the little change in slope of the string at the two ends of the small piece under consideration when determining the wave equation for that section of string by writing F=ma. Our rod model is a discredited variation in which the net force acting on a rod in the middle is determined by the marginally different slopes of the lines joining it to its neighbors. On the other hand, there is just one force acting on the final rod if it is free to move.

Its sole neighbor's force must be negligible and similar to the force differential between average neighbor rods for it to have the same acceleration as its neighbors. This indicates that the last curve formed by the dots on the rod ends (see equipment photo) must be almost horizontal because the final two rods are nearly lined up.

The edge of this image is a thread with progressively more rods grouped closer together. Hence, a string's free end boundary condition is that its slope must reach zero at the boundary.

When this boundary criterion is satisfied, it is easy to observe that a pulse will reflect with no change in sign. Just use the spreadsheet to send down two pulses of the same sign:

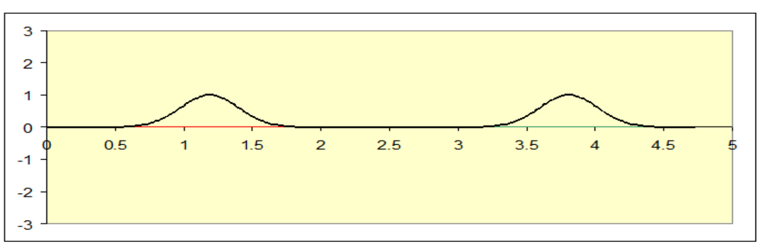


Figure 14.9

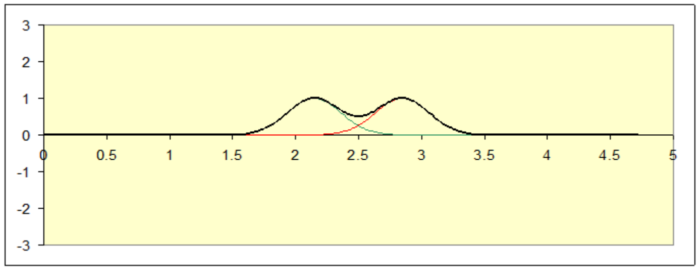


Figure 14.10

A pulse coming from the left will be reflected towards the end of the curve without changing in sign If we consider the left half to be a whole string with the right-hand end boundary constraint being that the string's slope be zero. This is because the complete symmetry makes it clear that the slope of the curve will always be zero at the central point.

As previously stated, while this may seem like an atypical boundary condition for a string, it turns out to be the ideal boundary condition for an organ pipe's open end, hence this approach has use in certain real-world systems.

14.9 Longitudinal Waves & Tuning Forks

Anything that vibrates produces sound waves. Every sound has a vibrating item at its source, whether it is a gossip between two persons, a piano, a violin, or the sound of a copy falling to the surface.

A helpful example of how a vibrating object can make sound is a tuning fork. The fork has two tines and a handle. The tines of the tuning fork vibrate when struck with a rubber hammer. The surrounding air molecules are disturbed by the tines' back and forth vibrating.

A high pressure area is created next to a tine when it extends outward from its normal position, compressing nearby air molecules into a small area of space. As the tine moves inward from its regular place, the air surrounding it expands, forming a region of low pressure adjacent to the tine. The high-pressure regions are called compressions, and the low-pressure areas are called rarefactions. As long as the tines vibrate, a pattern of low and high-pressure zones forms alternatively. These zones travel through the surrounding environment, conveying the sound signal from one point to another.

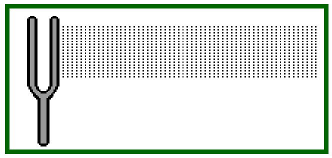


Figure 14.11

Sound can exist as a longitudinal wave or a transverse wave in solids. But in fluid media, sound waves can only travel in a longitudinal direction (such as gases and liquids).

A sound wave is shown as a longitudinal wave in the animation above. During a longitudinal wave, the medium's particle vibrations occur in the opposite direction from the direction of energy transmission.

The energy is seen in the animation above as leaving the tuning fork and moving from left to right. The molecules of air are oscillating from left to right and from right to left around a fixed point. This is the reason why a sound wave is longitudinal.

Another significant feature of waves is shown in the animation above. After closely examining the air particles (shown by dots), it can be seen that the molecules are pushed to the right before returning to their initial position on the left. The molecules of air are always oscillating around their initial position. The air molecules are not displaced in any net way. Air molecules always return to their original location after being momentarily disturbed from their resting state. This is to say, a sound wave, like any other wave, in which energy moves not matter from one to other place.

Summary

Understanding these boundary conditions is crucial for analyzing and designing systems involving transverse vibrations, as they dictate the possible modes of vibration and natural frequencies of the structure. By manipulating boundary conditions and material properties, engineers can tailor the vibrational behavior of bars and beams to suit specific applications, such as in musical instruments, structural elements, or mechanical systems. A tuning fork is a specialized bar with two prongs that vibrate transversely when struck. The boundary conditions are determined by how the prongs are supported or anchored, affecting the natural frequencies and mode shapes of vibration.

Keywords

Flexural vibrations, transverse waves, fixed boundary conditions

Objective type questions

1. Increasing mass will result in lower frequency.

   a) True

   b) False

2. When sound waves travel through air, the particles of air undergo:

   a) Longitudinal oscillations

   b) Transverse oscillations

   c) Circular oscillations

   d) No oscillations

3. When a tuning fork vibrates, it creates areas of:

a) High and low pressure alternately

b) High pressure only

c) Low pressure only

d) No pressure variation

4. Which type of waves are produced by tuning forks?

a) Transverse waves

b) Longitudinal waves

c) Surface waves

d) Electromagnetic waves

5. In a longitudinal wave, the particles of the medium oscillate:

a) Perpendicular to the direction of wave propagation

b) Along the direction of wave propagation

c) In random directions

d) In circular motions

Self Assessment

1. What is transverse and torsional vibration?
2. What is the equation for transverse vibration of a string?
3. Which tuning fork is used?
4. What is the principle of tuning fork?
5. When particles of the body vibrate parallel to its axis, then the body is said to be under?

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